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Optimization of parameters for a nuclear gas turbine plant

Beving, Duane U.
Massachusetts Institute of Technology

OPTIMIZATION OF PARAMETERS FOR A NUCLEAR GAS TURBINE PLANT

by

LIEUTENANT DUANE U. BEVING, U.S.N.

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF NAVAL ENGINEER
AND THE DEGREE OF
MASTER OF SCIENCE IN NAVAL ARCHITECTURE
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S. C. POWELL
THESIS SUPERVISOR

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DUANE U. BEVING, LIEUTENANT, UNITED STATES NAVY

B.S., U. S. NAVAL ACADEMY

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ABSTRACT

This thesis develops the mathematical expressions to evaluate component weight and space for all significant components within the containment vessel (secondary shielding) of a Gas Cooled Reactor (GCR) closed cycle gas turbine plant for ship propulsion. The equations are solved for a range of operating conditions using helium as the working fluid. Information from these equations was used to obtain the following:

1. Optimum total weight of components.
2. Curves of component weight.
3. Curves of component dimensions.

The resulting data support the following conclusions:

1. Optimum weight occurs at a pressure ratio that is lower than the value that gives best efficiency.

2. A heavy weight penalty must be paid for small increases in efficiency.

3. The regenerator is the controlling component in a cycle of this type. As heat transfer requirements go up (regenerator effectiveness approaches 100 %), regenerator weight rises sharply and limits appreciable gains in efficiency.

4. Rotating machinery length will have a greater influence on plant design than the weight of this component.

The curves should give the designer a rough idea of machinery component sizes and weights, and

are intended for use in the development and design of an efficient, nuclear gas turbine power plant.

Thesis Supervisor: S. C. Powell

Title: Associate Professor of Marine Engineering

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NOMENCLATURE

A	cross section or heat transfer area (ft^2)
C	absolute velocity (ft/sec) constant
Cp	specific heat at constant pressure (BTU/Lb$^{\circ}$R)
D	diameter (ft)
d	pitch diameter at mid-blade diameter
d_e	hydraulic diameter = $\frac{4(\text{cross section area})}{\text{wetted perimeter}}$, (ft)
d_x	equivalent outside flow diameter per tube (ft)
f	Fanning friction factor blade taper factor
g_o	gravitational constant
H	surface film coefficient of heat transfer (BTU/ft^2-HR-$^{\circ}$F) height (ft)
J	conversion factor (778 ft-Lb/BTU)
K	ratio of specific heats constant
k	thermal conductivity (BTU/ft-HR-$^{\circ}$F)
L	length (ft)
ℓ	blade length
M	mach number
m	mass (Lb)
N	number of tubes number of fuel rods number of stages
P	pressure (psia)
p	distance between tube centers (tube pitch), (ft)

Pr	Prandtl number
Q	heat rate (BTU/HR)
q	constant, function of tube arrangement
R	gas constant (ft-Lb/Lb$^{\circ}$R) radius (ft)
r	pressure ratio (>1)
Re	Reynolds number
S	entropy (BTU/Lb$^{\circ}$F)
T	temperature ($^{\circ}$R = $^{\circ}$F + 460) and $^{\circ}$F
U	heat transfer coefficient (BTU/ft^2-HR-$^{\circ}$F) circumferential velocity on pitch diameter
u	exponent = $\frac{K}{K-1}$
V	velocity (ft/sec) volume (ft^3)
v	exponent = $\frac{K-1}{K}$
W	work (BTU/Lb) weight (tons)
w	flow rate (Lb/HR)
Y	pressure loss factor = $\left(\frac{r_t}{r}\right)^v$
β	stress parameter = $\frac{\delta}{f\rho}$ (ft) angle of relative velocity
δ	blade stress (p si)
η	efficiency
η_x	regenerator effectiveness
Ω	angular velocity (rad/sec)

τ ratio of compressor isentropic exit temperature to its inlet temperature

ρ density (Lb/ft^3) or ($tons/ft^3$)

μ viscosity (Lb/HR-ft)

ΔT_m log mean temperature difference

Subscripts

A accumulator

a axial
area

c compressor
cold

d diameter

g gas

HPC high pressure compressor

HPT high pressure turbine

h hot

i inside

ic intercooler

LPC low pressure compressor

LPT low pressure turbine

ℓ length

m mean

N number of tubes

o outside

p piping

pc precooler

R	reactor
RG	regenerator
s	surface stage
t	turbine
V	reactor vessel
w	water weight
x	cross section

State Points

1.	precooler outlet LPC inlet
2.	LPC exit intercooler inlet
2s.	isentropic LPC exit
3.	intercooler outlet HPC inlet
4.	HPC exit cold side regenerator inlet
4s.	isentropic HPC exit
5.	cold side regenerator outlet reactor inlet
6.	reactor outlet HPT inlet
7.	HPT exit LPT inlet
7s.	isentropic HPT exit
8.	LPT exit hot side regenerator inlet
8s.	isentropic LPT exit
9.	hot side regenerator outlet precooler inlet

I. INTRODUCTION

The thermodynamic potential of the closed cycle gas turbine power plant has been well recognized for many years. Early attempts to effectively utilize this potential, however, were unsuccessful because of poor component efficiencies and the lack of a suitable high temperature heat source. Technical developments of high temperature gas turbines over the past years have enabled the designer to develop components with the necessary thermal efficiency, and the advent of the nuclear reactor has created the type of heat source necessary to drive the system.

In the design of a ship propulsion plant, certain parameters are not subject to the designer's control or choice. The maximum temperature in the cycle (T_6) may be restricted because of reactor fuel cladding creep strength limitations or the restriction imposed by the high pressure turbine (HPT) blading. The minimum temperature (T_1) of the working fluid is likely to be set from the sea water inlet temperature and reasonable size and weight coolers. A fixed reactor inlet temperature (T_5) is necessary to prevent cyclic thermal stresses from damaging the reactor structural material and moderator. In a gas cooled reactor, the inlet temperature is put at a relatively high value to anneal out radiation damage in the moderator. Compressor and turbine efficiencies

are dependent on the state of technical development of these machines. In view of the above, T_6, T_1, T_5, and rotating machinery efficiencies will most likely be fixed for any cycle because of mechanical design or metallurgical restraints and size of the plant.

The designer then has at his disposal other parameters which he must fix so as to give the most economic, efficient or the least weight and/or space propulsion plant.

The closed cycle nuclear gas turbine plant considered in this work consists of two principal sections: the reactor, which is located within the primary shield, and the propulsion equipment which is located within the secondary shielding or containment vessel along with the reactor. The main objective of this thesis was to develop a weight and space optimizing procedure for all significant components located within the containment vessel. The machinery components considered as significant with respect to the variation in their weights and sizes with changes in the cycle parameters are: the regenerator, coolers, rotating machinery, reactor, piping, and accumulator. The developed expressions for component weights and dimensions in terms of thermodynamic properties, gas constants and flow rate enables the designer to compare the relative size and weight of each component for different working fluids.

The second thesis objective was to make a weight and size calculation of these components using the proposed helium-cooled, graphite moderated, Maritime Gas-Cooled Reactor (MGCR) power plant as a reference. Solution of the developed equations for a range of operating conditions provided data for determining minimum weight and space for the regenerative, inter-cooled gas turbine cycle.

No gas turbine nuclear power plant has been or is presently being constructed for ship propulsion. However, with the technological advances being made in nuclear reactors, high temperature materials and rotating machinery, it is to be expected that a ship power plant of this type will someday be used. The shortage of component weight and space data for such a system will present a major problem to the ship preliminary designer who must develop an efficient nuclear gas turbine plant. The developed weight and space equations along with the system study of a particular plant undertaken in this thesis should assist the naval architect and marine engineer in developing the most economical, efficient or the least weight and/or space gas turbine nuclear power plant.

For any ship propulsion plant, it will be necessary for the ship designer to produce machinery components which give the best performance for the least

weight, cost, space or some other criteria. The submarine designer for instance would want to optimize his combination of machinery components in order to achieve the best performance for the least weight and space. The merchant ship designer on the other hand would direct his work toward producing a plant that gives the best performance for the least cost. Optimization of a propulsion plant can be performed for different reasons. Knowing the mission, size and requirements of the ship, the designer is able to optimize the machinery on a basis that gives the best power plant commensurate with the ship's purpose.

In an optimization procedure, the real problem is to reduce the number of variables so that they can be conveniently handled. The ideal situation is to have the weight or cost expression in terms of one parameter or at most two independent parameters. Computation work and time are greatly reduced with each less variable. Also desirable is an expression in closed analytic form which will greatly simplify calculations. However, the functions to be differentiated will usually not be found in this form, and the designer must then resort to a graphical, numerical or some other method of approximate differentiation. It is important that careful study be given all parameters in a weight or cost function to determine whether or not they are

independent. If two parameters are dependent, the method of evaluating the function will be different and generally more difficult than if all variables are independent.

The first step in eliminating the number of variables is to assign possible values to those quantities that are beyond the control of the designer. Secondly the concept of "balance" is used to reduce the number of variables. The latter step can be applied to components which bear a specific relationship to each other. For example, the task of finding the surface area of two different components for a given power plant might be reduced to a single problem of selecting a balanced combination of these areas. This method would be possible if the effects of each area on cost or plant weight were evidenced by their influences on the same working fluid property. The problem of selecting a balanced combination of areas is done separate from the optimization of the plant as a whole. All other components of the plant must be sensitive only to this fluid property and not to the various combinations of these two areas which establish that fluid property. Once a relationship between balanced components is formulated, the units comprising the balanced group are treated as a single component with a consequent elimination of variables.

In this work, a number of variables were eliminated

from consideration of step one. The "balance" concept was used to form a relationship between the compressors and the high pressure turbine. This step eliminated the high pressure turbine and low pressure turbine pressure ratios as variables. Thus once a reasonable value for the cycle pressure loss is specified, the remaining variables are flow rate (w), top pressure level (P), overall compressor ratio (r), regenerator effectiveness (η_x) and temperatures in the cycle. Both cycle efficiency and net work per pound of fluid flow are independent of top pressure as shown in Section II. Pressure is then of importance only in the way it effects weight and space of components. All remaining temperatures of the cycle are dependent on r. For a specific power output, flow rate is a function of the pressure ratio. With establishment of a fixed reactor inlet temperature as previously discussed, regenerator effectiveness is also a function of pressure ratio. Because the reactor inlet temperature is set, the cycle pressure ratio must always be less than the value that would make the turbine exhaust temperature equal to the reactor inlet temperature. For equal reactor inlet and turbine exhaust temperatures, a η_x of 100 per cent is required. A pressure ratio giving a turbine exhaust temperature lower than reactor inlet temperature would require a η_x greater than 100 per cent and hence presents

an impossible cycle.

In view of the above, there are two independent variables for a plant of this type. All weight and size equations were developed in terms of these two independent parameters: pressure and pressure ratio. Subsequent solution of the equations for a narrow range of pressure ratios indicated extreme variations in total weight and size of components.

The regenerator weight and size was found to dominate that of all other components by several orders of magnitude as regenerator effectiveness approaches 100 per cent. In order to achieve a higher efficiency without having to accept the large increase in weight contributed by the regenerator, consideration was given to the idea of using an additional piece of heat exchanger equipment between the regenerator and reactor inlet. A small quantity of high temperature fluid could be removed from the reactor to further heat the bulk of the fluid from the regenerator. The extraction of this fluid from the reactor might occur after it has passed up the outside of the reflector in cooling the thermal shield and vessel wall and before it flows along the surface of the fuel rod assemblies. Another possible extraction point for the small quantity of high temperature fluid might be in the LPT after partial expansion or at the HPT exhaust. This "bled" fluid would then be

recirculated to the additional heat exchanger to further raise the temperature of the bulk of the fluid coming from the regenerator. A considerable portion of the regenerator duty could then be taken over by the additional heat exchanger. The resulting lower regenerator effectiveness would lower regenerator weight. A study of such a proposal is needed to indicate if the reduction in regenerator weight, caused by a lower regenerator effectiveness, is more than the weight increase caused by the additional heat exchanger (with associated equipment) and the greater weights of other components (that might be caused by the slightly larger flow rate required in order to keep the same power output).

II. PROCEDURE

A. General

The basic procedure followed was:

1. Develop mathematical expressions for component weights and sizes in terms of state point properties and system constants.

2. Determine the required system constants.

3. Solve the equations for a range of operating conditions.

Steps 1 and 2 are carried out in this section with the detailed procedures of the development performed in Appendices A, B and C. Step 3 is found in Appendix D.

The plant components considered, consist of the regenerator, precooler, intercooler, reactor, accumulator, piping, and rotating machinery comprised of the low pressure compressor (LPC), high pressure compressor (HPC), high pressure turbine (HPT) and low pressure turbine (LPT). The HPT drives the HPC and LPC, and the LPT provides shaft power. A low pressure turbine bypass heat sink was not considered as a plant component. Preliminary studies [13]* indicate that direct bypassing of the low pressure turbine without an additional

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*Numbers in brackets refer to references listed in Appendix E.

heat exchanger is feasible. With this system thermal cycling conditions appear to be within the capabilities of the different component materials. A system requiring additional piping, but still no bypass heat sink, would use a split-flow scheme. A portion of the bypassed fluid would be piped directly to the precooler and the remainder would proceed through the regenerator as usual. The reactor inlet temperature could be kept more closely regulated with this latter method. Figure II-1 is a flow diagram of the system indicating state points and some of the symbols used in the development of the equations. Figure II-2 is the corresponding temperature-entropy diagram for the cycle.

The mathematical development that follows is based on the perfect gas relationships and on turbulent flow of the working fluid in ducts and piping. In addition, the velocity of fluid flow in the system is considered to be of the order of magnitude where static pressure and temperature are approximately equal to stagnation conditions. All gas properties are evaluated at the mean fluid temperature in a particular component and are considered constant over the range of temperatures encountered in that component

For minimum compressor work in a two-stage compression with intercooling to the low temperature of the cycle (T_1), the pressure ratio of each compressor

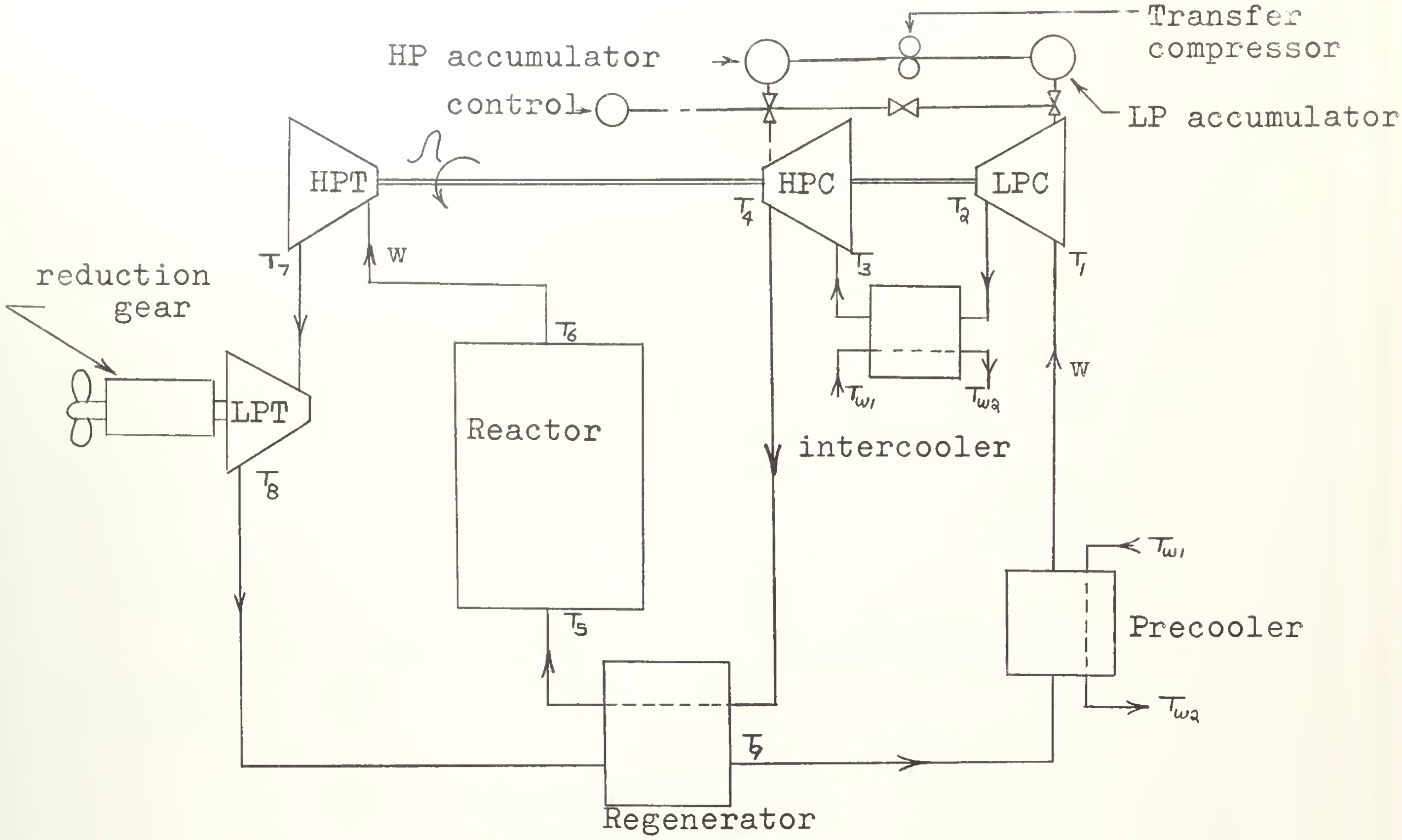
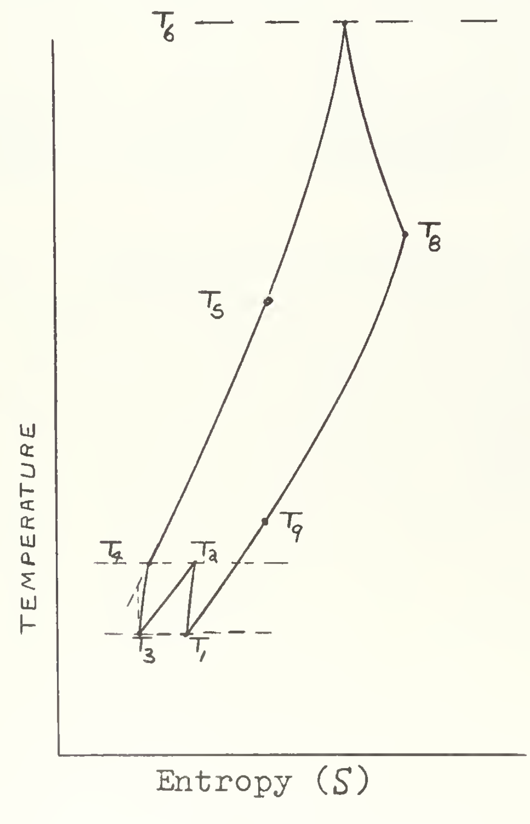


Figure II~~I~~-1
Flow Diagram for helium-cooled, graphite moderated
Nuclear Gas Turbine Power Plant

Figure II~~I~~-2
Temperature-Entropy Diagram



can be readily shown to equal the square root of the over-all pressure ratio:

$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = \left(\frac{P_4}{P_1}\right)^{.5} \qquad (2\text{-}1)$$

The above condition also implies negligible pressure drop of the fluid through the intercooler.

Total compressor work can be written for equal compressor efficiencies as:

$$W_c = \frac{2C_pT_1}{\eta_c}\left(\tau^{.5} - 1\right) \qquad (2\text{-}2)$$

Total turbine work for equal efficiencies is:

$$W_t = \eta_t C_p T_6 \left(1 - \frac{1}{Y\tau}\right) \qquad (2\text{-}3)$$

Y being the pressure loss factor in the cycle defined as:

$$Y = \frac{T_6/T_{8s}}{T_{4s}/T_1} = \frac{\tau_t}{\tau} = \left(\frac{r_t}{r}\right)^V \approx \left(\frac{P_6}{P_8}\,\frac{P_3}{P_4}\,\frac{P_1}{P_2}\right)^V = \left(\frac{P_6}{P_5}\,\frac{P_5}{P_4}\,\frac{P_9}{P_8}\,\frac{P_1}{P_9}\,\frac{P_3}{P_2}\right)^V \qquad (2\text{-}4)$$

τ is the ratio of compressor isentropic exit temperature to its inlet temperature and τ_t is the ratio of turbine inlet temperature to its isentropic exit temperature. The factors in the last term are the fluid pressure drops contributed respectively by the reactor, cold side of regenerator, hot side of regenerator, precooler and inter-cooler. The pressure drop being a function of working

fluid, pressure, and component arrangement and geometry is usually estimated by the designer for each component. The total cycle pressure drop and hence Y, for the cycle or a component, can be determined.

The net work for the cycle is simply $W_t - W_c$. Using (2-2) and (2-3) the net work (W_{net}) per pound of fluid flow can be written as:

$$\frac{W_{net}}{C_p T_1} = \eta_t \frac{T_6}{T_1} \left(1 - \frac{1}{Y\tau}\right) - \frac{2}{\eta_c} \left(\tau^{.5} - 1\right) \qquad (2\text{-}5)$$

Heat into the cycle from the reactor can be written as:

$$\frac{Q_{in}}{C_p T_1} = \frac{C_p (T_6 - T_5)}{C_p \; T_1} = \frac{T_6 - T_5}{T_1} \qquad (2\text{-}6)$$

The thermal efficiency of the cycle is then:

$$\eta = \frac{W_{net}}{Q_{in}} \qquad (2\text{-}7)$$

Combining (2-5), (2-6) and (2-7):

$$\eta = \frac{\eta_t \frac{T_6}{T_1} \left(1 - \frac{1}{Y\tau}\right) - \frac{2}{\eta_c} \left(\tau^{.5} - 1\right)}{\frac{T_6 - T_5}{T_1}} \qquad (2\text{-}8)$$

For reasons discussed in Section I, T_6, T_5, T_1, η_t and η_c are not generally subject to the designer's

control. With a reasonable value assigned to Y, the net work and efficiency become functions of cycle pressure ratio only. By setting the derivative with respect to τ equal to zero in (2-5), the optimum value of cycle pressure ratio (r) is found which maximizes cycle net work. Similarly the optimum value of r can be found from (2-8) which gives maximum cycle efficiency.

B. Heat Exchanger Equipment

All heat exchanger equipment considered is assumed to behave as if of the shell and tube counterflow configuration. In the regenerator the high pressure, cold gas flows inside the tube, with the hotter gas flowing longitudinally around the tube interspace. The increased strength of the tubes with internal pressure would necessitate this arrangement. Water flows through the tubes and gas around the tubes in both coolers. This arrangement would facilitate the required periodic cleaning of the water side of the coolers. The precooler and intercooler require large cooling water flows with low water velocities and low pump heads. Design factors for the coolers follow marine steam condenser practice.

1. Regenerator

For a given gas, tube size and tube arrangement, equations (A-30)* and (A-31) can be written respectively

*Equations carrying letter designations are from the corresponding appendix.

as:

$$Nd_i^2 = K_{Nd}\ w\left(\frac{\eta_x}{1-\eta_x}\right)^{.5} \frac{1}{P}\ (T_c + r^2\ T_h\ C_d)^{.5} \qquad (2\text{-}9)$$

$$Nd_i L = K_{NdL}\ w\left(\frac{\eta_x}{1-\eta_x}\right)^{1.4} \frac{1}{P^{.8}}\ (T_c + r^2 T_h C_d)^{.4} \qquad (2\text{-}10)$$

where $C_d = \frac{1}{q^3}\left(\frac{d_i}{d_o}\right)^{4.8}$ and the K's are constants of proportionality. Since T_c and T_h were defined as

$$T_c = \frac{T_5 + T_4}{2} \qquad (2\text{-}11)$$

and $$T_h = \frac{T_8 + T_9}{2} \qquad (2\text{-}12)$$

they may be put in terms of pressure ratio and known cycle temperatures. From the definition of turbine work:

$$C_p(T_6 - T_8) = C_p\eta_t(T_6 - T_{8s})$$

$$T_8 = T_6[1 - \eta_t(1 - \frac{1}{Y\tau})] \qquad (2\text{-}13)$$

Using (2-13) and (A-4), (2-11) and (2-12) can be written:

$$T_c = \frac{T_5(2-\eta_x) - \eta_x\ T_6[1 - \eta_t(1 - \frac{1}{Y\tau})]}{2\ (1 - \eta_x)} \qquad (2\text{-}14)$$

$$T_h = \frac{T_6[1-\eta_t(1-\frac{1}{Y\tau})](2-3\eta_x) + T_5\eta_x}{2(1-\eta_x)} \tag{2-15}$$

Substituting (2-14) and (2-15) into (2-9) and (2-10), cross sectional and heat transfer area variations become functions of presure ratio (r), η_x, w, P and known quantities: For a specified maximum and minimum temperature of the working fluid (T_6 and T_1), a fixed reactor inlet temperature (T_5) and an appropriate value for turbine efficiency, the area variations are dependent on r, P, and w. η_x becomes known once the pressure ratio is specified, since reactor inlet temperature is fixed.

Regenerator weight may be written as a function of the heat transfer area:

$$W_{RG} = K_1(Nd_iL) + K_2 \tag{2-16}$$

where K_1 and K_2 are constants and (Nd_iL) is given by (2-10) with (2-14) and (2-15) substituted for T_c and T_h.

2. Coolers

For a given gas, tube size and tube arrangement, equations (A-32) and (A-33) for the precooler become:

$$Nd_o^2 = K_{Nd}\ w\left(\frac{\Delta T_g}{\Delta T_m}\right)^{.5} \quad T_{pc}^{.5} \quad \frac{r}{P} \tag{2-17}$$

$$Nd_oL = K_{NdL}\, w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{1.4} \left(T_{pc}\right)^{.4} \left(\frac{r}{P}\right)^{.8} \tag{2-18}$$

where $\Delta T_g = T_9 - T_1$ (2-19)

$$T_{pc} = \frac{T_9 + T_1}{2} \tag{2-20}$$

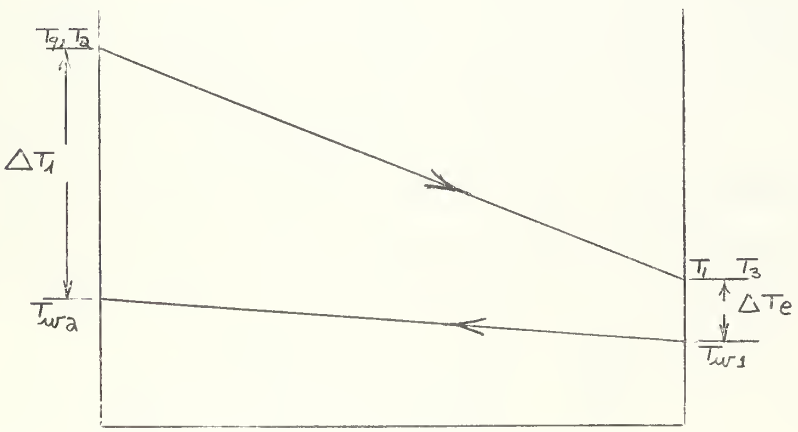
Using equations (2-13) and (A-4), T_9 can be expressed as:

$$T_9 = \frac{T_6(1-2\eta_x-\eta_t+2\eta_t\eta_x + \frac{\eta_t}{Y\tau} - \frac{2\eta_t\eta_x}{Y\tau}) + \eta_x T_5}{1 - \eta_x} \tag{2-21}$$

ΔT_m, the LMTD may be written as:

$$\Delta T_m = a(\Delta T_A) = a\left(\frac{\Delta T_i + \Delta T_e}{2}\right) = a[T_{pc} - T_{mw}] \tag{2-22}$$

where ΔT_A is the arithmetic mean temperature difference and "a" is dependent on the ratio $\frac{\Delta T_i}{\Delta T_e}$; see Figure II-3.



Z - distance along cooler

Figure II-3

Cooler Temperature-Length Diagram

For the precooler and intercooler, a mean water temperature (T_{mw}) of 80°F is assumed, based on 70°F sea water inlet temperature and a 20°F water temperature rise through each cooler. From [2], values of "a" range from 1.00 to 0.710 as $\frac{\triangle T_i}{\triangle T_e}$ goes from 1 to 10.

With T_{w1} and T_{w2} specified, "a" can be approximated by:

$$a = 1.227 - .0006\, T_9 \tag{2-23}$$

for the range of gas temperatures likely to be encountered at the precooler inlet.

With the above simplifications, (2-19), (2-20) and (2-22) may be expressed in terms of pressure ratio, η_x and known temperatures and efficiencies by using (2-21) and (2-23). The resulting relations for (2-19), (2-20) and (2-22) may then be substituted into (2-17) and (2-18) giving rather long equations for the cross sectional and heat transfer areas in terms of flow rate, pressure, pressure ratio, η_x and known quantities.

For a given working fluid, tube size and tube arrangement, (A-34) and (A-35) for the intercooler become:

$$Nd_o^2 = K_{Nd}\, w\left(\frac{\triangle T_g}{\triangle T_m}\right)^{.5} \left(T_{ic}\right)^{.5} \frac{r^{.5}}{P} \tag{2-24}$$

$$Nd_o L = K_{NdL}\, w\left(\frac{\triangle T_g}{\triangle T_m}\right)^{1.4} \left(T_{ic}\right)^{.4} \frac{r^{.4}}{P^{.8}} \tag{2-25}$$

where $\Delta T_g = T_2 - T_3 = T_2 - T_1$ (2-26)

$$T_{ic} = \frac{T_2 + T_3}{2} = \frac{T_2 + T_1}{2} \tag{2-27}$$

T_2 represents the temperature out of the LPC and can be expressed from compressor work as:

$$T_2 = T_1 \left[\frac{1}{\eta_c} (\tau^{.5} - 1) + 1 \right] \tag{2-28}$$

Combining (2-28) with (2-26) and (2-27):

$$\Delta T_g = T_1 \left[\frac{1}{\eta_c} (\tau^{.5} - 1) \right] \tag{2-29}$$

$$T_{ic} = \frac{T_1}{2} \left[\frac{1}{\eta_c} (\tau^{.5} - 1) + 2 \right] \tag{2-30}$$

The LMTD corresponds to that of the precooler:

$$\Delta T_m = a(T_{ic} - T_{mw}) \tag{2-31}$$

where "a" can be approximated by:

$$a = 1.814 - .0014\ T_2 \tag{2-32}$$

for reasonable values of gas temperature at the intercooler inlet. Substituting (2-28) into (2-32):

$$a = 1.814 - .0014\ T_1 \left[\frac{1}{\eta_c} (\tau^{.5} - 1) + 1 \right] \tag{2-33}$$

Combining equations (2-30), (2-31) and (2-33):

$$\triangle T_m = \left(1.81-.0014\ T_1[\frac{1}{\eta_c}(\tau^{.5}-1)+1]\right)\left(\frac{T_1}{2}[\frac{1}{\eta_c}(\tau^{.5}-1)+2]-540\right) \quad (2\text{-}34)$$

where T_{mw} was taken as $80 + 460 = 540^{\circ}R$.

By placing (2-29), (2-30) and (2-34) into both (2-24) and (2-25), the two intercooler area variations are expressed in terms of flow rate, pressure, pressure ratio and known temperatures and compressor efficiency.

As for the regenerator, the precooler and intercooler weights may be expressed as functions of the heat transfer area. For the precooler the basic weight equation is:

$$W_{pc} = K_1\ w\left(\frac{\triangle T_g}{\triangle T_m}\right)^{1.4} T_{pc}^{\ .4} \left(\frac{r}{P}\right)^{.8} + K_2 \qquad (2\text{-}35)$$

where $\triangle T_g$, $\triangle T_m$ and T_{pc} can be expressed in terms of P, r, η_x and known quantities from equations (2-19) through (2-23).

The weight equation for the intercooler:

$$W_{ic} = K_1\ w\left(\frac{\triangle T_g}{\triangle T_m}\right)_{ic}^{1.4} \left(T_{ic}\right)^{.4} \frac{r^{.4}}{P^{.8}} + K_2 \qquad (2\text{-}36)$$

where $\triangle T_g$, $\triangle T_m$ and T_{ic} can be expressed in terms of P, r and known values from equations (2-29), (2-30) and (2-34).

C. Reactor

The reactor weight and size equations are developed by considering the reactor as another piece of heat exchanger apparatus. The proposed MGCR helium cooled, graphite moderated reactor with a heterogeneous assembly of fuel rods [11], [12], [14] is used as the reference reactor.

For a constant heat generation flux along the coolant channel and radially over the reactor, the gas temperature rises with a constant slope versus coolant channel length. The coolant wall temperature will also rise with the same slope from $(T_5 + \triangle T)$ to $(T_6 + \triangle T)$ where $\triangle T$ is the coolant wall temperature minus the temperature of the coolant. The maximum wall and coolant temperature will occur at the reactor outlet with the outlet coolant temperature in each channel being equal for uniform heat generation in the core. A constant heat generation flux is substantially true for a relatively small reactor core using good reflector material [1], [3], [15] and [18].

The heat transfer in the reactor can be represented by:

$$Q = H A_s \triangle T \tag{2-37}$$

$$Q = w C_p(T_6 - T_5) \tag{2-37A}$$

with $\triangle T$ = maximum wall temperature minus the outlet

temperature of the coolant. H is given by (A-13):

$$H = \frac{C_1 \quad k^{.2} \; (w \; C_p)^{.8}}{N^{.8} \; D^{1.8} \; (Pr)^{.4}} \tag{2-38}$$

D and N represent the outside diameter of the fuel rod and number of fuel rods respectively. The pressure drop through the reactor is given by (A-17). The term $\frac{1}{2}\left(\frac{T_w - T}{T}\right)$ in (A-17) can not be neglected, but is equal to a constant for uniform heat generation and fixed inlet and outlet coolant temperatures. (A-17) then becomes:

$$\ell n \frac{P_6}{P_5} = -C_2 \; \frac{K_M^{\;2}}{2} \quad [4f \; \frac{L}{D}] \tag{2-39}$$

where for low mach numbers, static pressures and temperatures can be interchanged with stagnation properties. The pressure drop factor (Y_R) for the reactor is:

$$Y_R = \left(\frac{P_6}{P_5}\right)^V \tag{2-40}$$

Following the procedure used in obtaining equation (A-22) in Appendix A, the pressure drop term for the reactor can be written:

$$y_R = C_3 \; \frac{K-1}{2} \; M^2 \; (4f \; \frac{L}{D}) \tag{2-41}$$

C_3 being a constant determined by the fixed inlet and

outlet temperatures and the maximum wall temperature of the reactor. Using (A-24) and (A-26) the Fanning friction factor and mach number squared are represented respectively by:

$$f = .0438 \left(\frac{DNC_p \mu}{w \; C_p} \right)^{.2} \tag{2-42}$$

$$M^2 = \left(\frac{4J}{\pi} \right)^2 \frac{1}{g_o} (wC_p)^2 \frac{T_{mR}}{RK} \left(\frac{K-1}{K} \frac{1}{PND^2} \right)^2 \tag{2-43}$$

$T_{mR} = \dfrac{T_6 + T_5}{2}$ = constant. Combining all constant terms:

$$M^2 = C_M \frac{(wC_p)^2}{KR} \left(\frac{K-1}{K} \frac{1}{PND^2} \right)^2 \tag{2-44}$$

Substitute (2-42) and (2-44) into (2-41):

$$y_R = C_{M3} \left(\frac{K-1}{K} \right)^3 \frac{(wC_p)^{1.8}}{R} \frac{L}{P^2} \frac{(\mu C_p)^{.2}}{N^{1.8} D^{4.8}} \tag{2-45}$$

Combine (2-37) and (2-38) and write:

$$\frac{Q}{\Delta T} = \frac{C_i \; k^{.2} \; (wC_p)^{.8}}{N^{.8} D^{1.8} (Pr)^{.4}} (\pi NDL) \tag{2-46}$$

ΔT as defined with equations (2-37) is a fixed value and so (2-46) can be written:

$$C_1 = \frac{Q \; D^{.8} \; (Pr)^{.4}}{k^{.2} (wC_p)^{.8} N^{.2} L} \tag{2-47}$$

Multiplying equation (2-45) by (2-47) and solving the resulting expression for ND^2:

$$ND^2 = C_{ND} \left(\frac{K-1}{K}\right)^{1.5} \left(\frac{wC_pQ}{R}\right)^{.5} \frac{(Pr)^{.3}}{P} \qquad (2\text{-}48)$$

(2-37A) can be substituted for Q to give:

$$ND^2 = C_{ND} \left(\frac{K-1}{K}\right)^{1.5} wC_p \frac{(Pr)^{.3}}{R^{.5}} \frac{(T_6-T_5)^{.5}}{P} \qquad (2\text{-}49)$$

The above equation represents the cross sectional area variation of the reactor. For fixed inlet and outlet temperatures and a given fuel element and coolant channel configuration, it is dependent on the flow rate and top pressure level in the cycle.

The corresponding heat transfer area (NDL) equation is obtained by solving (2-47) for L, dividing by D, making the appropriate substitution for $(ND^2)^{.2}$ and multiplying this result with (2-49):

$$NDL = C_{NDL} \left[\left(\frac{K-1}{K}\right)^{1.2} \left(\frac{1}{R}\right)^{.4} (Pr)^{.64} \left(\frac{D}{k}\right)^{.2} C_p\right] w \left(\frac{1}{P}\right)^{.8} \left(T_6-T_5\right)^{1.4} \qquad (2\text{-}50)$$

In (2-50), L is the coolant channel length or the core height, H. Using the bare core approximation, the relationship between height and radius for a critical cylinder of minimum volume is given by reference [4] as:

$$H = \frac{\pi\ (2)^{.5}}{2.405} R = 1.85R \qquad (2\text{-}51)$$

D is fixed for a given reactor and N, the number of fuel rods (coolant channels) is a function of reactor cross sectional area: $N = C_N R^2$. Placing this and (2-51) into (2-50), reactor radius cubed becomes:

$$R^3 = C_R \left[\left(\frac{K-1}{K}\right)^{1.2} \left(\frac{1}{R}\right)^{.4} (Pr)^{.64} \left(\frac{1}{D}\right)^{.8} \left(\frac{1}{k}\right)^{.2} C_p \right] \frac{w}{P^{.8}} (T_6 - T_5)^{1.4} \tag{2-52}$$

Reactor weight can be expressed as:

$$W_R = \rho_R \pi R^2 H \tag{2-53}$$

where ρ_R is the overall reactor density determined from a reference or similar reactor. Using (2-51) and (2-52) reactor weight becomes:

$$W_R = C_R \left[\left(\frac{K-1}{K}\right)^{1.2} \left(\frac{1}{R}\right)^{.4} (Pr)^{.64} \left(\frac{1}{D}\right)^{.8} \left(\frac{1}{k}\right)^{.2} C_p \right] \frac{w}{P^{.8}} (T_6 - T_5)^{1.4} \tag{2-54}$$

For a given coolant and fuel element configuration it is:

$$W_R = K_R \frac{w}{P^{.8}} (T_6 - T_5) \tag{2-55}$$

The R and H used above are for the core and do not represent the actual size of the reactor. The overall reactor radius and height, exclusive of shielding, can be taken as some constant multiple of the core dimensions for small changes in core R and H. From the MGCR reference reactor, the reactor vessel radius (R_V) and height (H_V) can be represented as:

$$R_V = 1.63\ (R) \tag{2-56}$$

$$H_V = 3.31(H) \tag{2-57}$$

D. Turbo Machinery

1. Compressors

In formulating length and weight equations for the turbines and compressors, it was assumed that the length of each component varies directly with the number of stages and that weight is proportional to the product of annulus area at the low pressure end and the length. Therefore by picking reasonable values for: efficiency, β, $\frac{C_a}{U}$, β_1, β_2 and $\frac{\ell}{d}$, equation (B-19) in Appendix B can be written as:

$$N_{LPC} = K\ T_1\ (\tau^{.5} - 1) \tag{2-58}$$

Since T_1, the minimum temperature of the cycle is fixed, the LPC length is:

$$L_{LPC} = K_\ell\ (\tau^{.5} - 1) \tag{2-59}$$

The annulus area given by (B-12) becomes:

$$A_{LPC} = K_a\ \frac{wr}{P} \tag{2-60}$$

w is the flow rate, r the pressure ratio and P the top pressure in the cycle.

The weight equation, after applying the condition stated above to (2-59) and (2-60), is:

$$W_{LPC} = K_w \frac{wr}{P} (\tau^{.5} - 1) \qquad (2\text{-}61)$$

Using (B-11) and (B-15), the tip diameter at the LPC low pressure end becomes:

$$D_{LPC} = \left(1 + \frac{\ell}{d}\right)\left(\frac{wr}{P}\right)^{.5} = K_d \left(\frac{wr}{P}\right)^{.5} \qquad (2\text{-}62)$$

The four corresponding equations for the HPC are indicated below:

$$L_{HPC} = K_\ell (\tau^{.5} - 1) \qquad (2\text{-}63)$$

$$A_{HPC} = K_a \frac{w\ r^{.5}}{P} \qquad (2\text{-}64)$$

$$W_{HPC} = K_w \frac{w\ r^{.5}}{P} (\tau^{.5} - 1) \qquad (2\text{-}65)$$

$$D_{HPC} = K_d \left(\frac{w}{P}\right)^{.5} r^{.25} \qquad (2\text{-}66)$$

The constants for each of the above equations are not necessarily applicable to both the HPC and the LPC. Each machine has a particular constant associated with it.

2. Turbines

As for the compressors, reasonable values of efficiences, β, $\frac{C_a}{U}$, β_1, β_2 and $\frac{\ell}{d}$ can be assumed or obtained from the turbine manufacturer. These factors will change very little over a wide range of operating conditions

(pressure, pressure ratio, flow rate, etc.) and so the size and weight equations can be expressed similar to those for the compressors.

For the HPT, (B-32) gives the number of stages as:

$$N_{HPT} = K\ (\tau^{.5} - 1) \tag{2-67}$$

Turbine length, annulus area, weight and tip diameter equations become respectively upon using the basic relations derived in Section III of Appendix B:

$$L_{HPT} = K_{\ell}\ (\tau^{.5} - 1) \tag{2-68}$$

$$A_{HPT} = K_a \quad \frac{w}{P}\ \frac{[1 - \eta_t\ C(\tau^{.5} - 1)]}{[1 - C(\tau^{.5} - 1)]^{u}} \tag{2-69}$$

where $C = \dfrac{2\ T_1}{T_6 \eta_t \eta_c}$

$$W_{HPT} = K_w \quad \frac{w}{P}\ \frac{(\tau^{.5} - 1)[1 - \eta_t C(\tau^{.5} - 1)]}{[1 - C(\tau^{.5} - 1)]^{\mu}} \tag{2-70}$$

$$D_{HPT} = K_d \left(\frac{w}{P}\right)^{.5} \left(\frac{[1 - \eta_t\ C(\tau^{.5} - 1)]}{[1 - C(\tau^{.5} - 1)]^{\mu}}\right)^{.5} \tag{2-71}$$

For the LPT, the equations are:

$$L_{LPT} = K_{\ell}\ \frac{[1-\eta_t C(\tau^{.5}-1)][Y\tau(1-C[\tau^{.5}-1])-1]}{Y\tau\ [1 - C(\tau^{.5} - 1)]} \tag{2-72}$$

$$A_{LPT} = K_a \frac{wr}{P} [1 - \eta_t(1 - \frac{1}{Y\tau})] \tag{2-73}$$

$$W_L = K_w \frac{wr}{P} \frac{[1-\eta_t(1-\frac{1}{Y\tau})][1-\eta_t C(\tau^{.5}-1)][Y\tau(1-C(\tau^{.5}-1))-1]}{Y\tau[1-C(\tau^{.5}-1)]} \tag{2-74}$$

$$D_{LPT} = K_d \left(\frac{wr}{P}\right)^{.5} [1 - \eta_t(1 - \frac{1}{Y\tau})]^{.5} \tag{2-75}$$

The constants associated with each equation will be different for the HPT and LPT unless considerable similarity exists between the two machines.

E. PIPING AND ACCUMULATOR

1. General

Both piping and the accumulator contribute significantly toward the total weight of the components within the containment vessel. In addition, the space occupied by the accumulator will effect the size of the containment vessel and hence the amount of secondary shielding.

Weight equations for piping and the accumulator are developed in this section. The constants associated with these equations were not determined due to lack of suitable data from a reference design. The equations however indicate the trends to be expected for different operating conditions.

2. Piping

Weight of piping is considered proportional to the internal surface area plus a constant.

$$W_p = K_1 \pi DL + K_2 \qquad (2\text{-}76)$$

For negligible heat transfer, the pressure drop term for the piping is expressed as:

$$y_p = \frac{K-1}{2} f M^2 \frac{L}{D} \qquad (2\text{-}77)$$

From (A-24), the Fanning friction factor for turbulent flow is:

$$f = .044 \left(\frac{DN\mu C_p}{wC_p} \right)^{.2} \qquad (2\text{-}78)$$

Mach number squared is given by (A-26) as:

$$M^2 = C \frac{(wC_p)^2}{R} \frac{T}{P^2} \frac{(K-1)^2}{K^3} \frac{1}{D^4} \tag{2-79}$$

Substitute (2-78) and (2-79) into (2-77) and solve resulting expression for D.

$$D = K_p \frac{(\mu C_p)^{.041}}{P^{.41}} (wC_p)^{.375} \left(\frac{TL}{R}\right)^{.21} \left(\frac{K-1}{K}\right)^{.62} \tag{2-80}$$

For a given working fluid, (2-80) becomes:

$$D = K_p \frac{w^{.375}}{P^{.41}} (TL)^{.21} \tag{2-81}$$

Since temperature in equation (2-81) is raised to only the 0.21 power, temperature does not effect the diameter (weight) significantly. As a preliminary conservative weight estimate, the maximum temperature in the cycle is used with (2-81). For greater accuracy, the piping length could be divided into two halves, a hot and a cold section. Equation (2-81) can now be written as:

$$D = K_p \frac{w^{.375}}{P^{.41}} (L)^{.21} \tag{2-82}$$

Combining (2-76) and (2-82):

$$W_p = K_p \frac{w^{.375}}{P^{.41}} L^{1.21} + K_2 \tag{2-83}$$

Considering the length of piping within the containment vessel as a fixed quantity for similar ships, (2-83) becomes:

$$W_p = K_1 \frac{w^{.375}}{P^{.41}} + K_2 \tag{2-84}$$

The constants in (2-84) were not determined since suitable information from a reference design concerning length and weight of piping was not available.

3. Accumulator

The accumulator weight is expressed as a function of the total weight of the working fluid in the cycle and the allowable stress in the wall material. Considering a long cylindrical container, the surface area, upon neglecting the end plates, is:

$$A = 2\pi RL \tag{2-85}$$

The volume of the working fluid is given by:

$$V = \pi R^2 L \tag{2-86}$$

$$V = m \frac{RT}{P} \tag{2-87}$$

where m represents the total mass of the working fluid within the cycle. Weight of the accumulator is simply the surface area times the product of wall thickness and material density.

$$W_A = 2\pi RLt\rho \tag{2-88}$$

Using simple hoop stress analogy, wall thickness becomes:

$$t = \frac{(P-P_a)R}{\sigma} \tag{2-89}$$

where P_a is the external (atmospheric) pressure and P is the pressure of the fluid within the accumulator. For rapid changes of load, the pressure of the fluid within the accumulator must be greater than the top pressure of the cycle. Equation (2-87) indicates a high pressure is also desirable from the standpoint of reducing the size of the container. To facilitate rapid changes in plant load [10], the accumulator is arranged in two parts: a high and low pressure section. When reduction in loads occur, gas is bled from the HPC discharge to a low pressure accumulator or receiver. As the low pressure flask fills, it is emptied by a piston type compressor which compresses the gas to a pressure considerably above the top pressure in the cycle. The small piston compressor discharges to the HP accumulator. As loads again increase, the HP accumulator discharges into the high pressure line between the HPC and the regenerator. Equating (2-86) and (2-87) and solving for the radius squared (R^2):

$$R^2 = \frac{mRT}{\pi LP} \tag{2-90}$$

Combining (2-88), (2-89) and (2-90), the weight of accumulator becomes:

$$W_A = \frac{2\rho mRT}{\delta} \frac{(P-P_a)}{P} \tag{2-91}$$

Since the temperature of the gas to be put in the accumulator will be of the order of 200 - 300°F (the HPC discharge gas temperature), the accumulator material need only be a carbon steel. Therefore for a given gas and a moderate storage temperature, equation (2-91) reduces to:

$$W_A = K \frac{m\,(P-P_a)}{\delta\,P} \tag{2-92}$$

Because the pressure within the accumulator will be several orders of magnitude larger than atmospheric pressure, $\frac{P-P_a}{P} \approx 1$ and (2-92) becomes:

$$W_A = K \frac{m}{\delta} \tag{2-93}$$

Maintaining a constant hoop stress for a fixed cylinder radius means that the pressure to wall thickness ratio must be a constant. With these restrictions, accumulator weight is written:

$$W_A = K_A\, m \tag{2-94}$$

F. Evaluation of Constants

The mathematical development of the weight and space equations necessitated the introduction of a number of unknown constants. Some constants depend upon materials used, some are proportionality constants and others are constants of design.

To solve the equations, it is necessary to evaluate each constant. Accurate evaluation of the constants requires a thorough investigation of similar plants, with note taken of the specifications, weight statements, heat balances and other information.

The equations developed under Section II and Appendices A and B are general in nature and not restricted to any particular working fluid or type of ship. Only in the evaluation and application of the constants does the problem become one of a specific type or design.

Since no shipboard nuclear closed cycle gas turbine plant exists, available data for evaluating the constants was limited. The author relied mainly on information from the proposed MGCR propulsion plant presented in references [11], [12], [14], [15] and, where applicable on information from existing open cycle gas turbine plants. The heat balance, specifications, and weight estimates for the helium-cooled, graphite moderated MGCR plant allowed satisfactory

evaluation of the constants for all components except piping and the accumulator.

The evaluation of these constants and sample calculations are given respectively in Appendices C and D.

III. RESULTS

A. Cycle Conditions

Original results were obtained in the form of data from hand calculations of the developed equations with reference to the helium cooled, graphite moderated MGCR power plant. Calculations were made by varying the compressor pressure ratio (r) for two different reactor inlet temperatures (T_5): $T_5 = 700^{\circ}F$ and $T_5 = 791^{\circ}F$.

Table III-A lists the fixed values used for both sets of calculations.

Table III-A

Fixed Cycle Conditions

LPT (net power) output	=21,500HP	Pressure drop=$\frac{r_t}{r}$	=.86
Working Fluid	=Helium	Coolant Water Inlet Temperature(T_{w1})	=$70^{\circ}F$
Top Pressure (P)	=1000 psia	Coolant Water Outlet Temperature(T_{w2})	=$90^{\circ}F$
Maximum Cycle Temperature (T_6)	=$1300^{\circ}F$	Turbine Efficiency (η_t)	=.90
Minimum Cycle Temperature (T_1) = T_3	= $100^{\circ}F$	Compressor Efficiency (η_c)	=.86

B. Graphs

The data is presented in graphical form in accordance with Table III-B.

Table III-B

Graphs

Plot	Figure
Total Weight and Efficiency vs Pressure Ratio	III-1
Regenerator Weight vs Pressure Ratio	III-2
Precooler and Intercooler Weight vs Pressure Ratio	III-3
Reactor Weight vs Pressure Ratio	III-4
Rotating Machinery Weight vs Pressure Ratio	III-5
Regenerator Dimensions vs Pressure Ratio	III-6
Precooler Dimensions vs Pressure Ratio	III-7
Intercooler Dimensions vs Pressure Ratio	III-8
Reactor Dimensions vs Pressure Ratio	III-9
Rotating Machinery Length and Low Pressure Turbine Exhaust Diameter vs Pressure Ratio	III-10
Flow Rate (lb/HP-HR) vs Pressure Ratio	III-11

Figure III-1

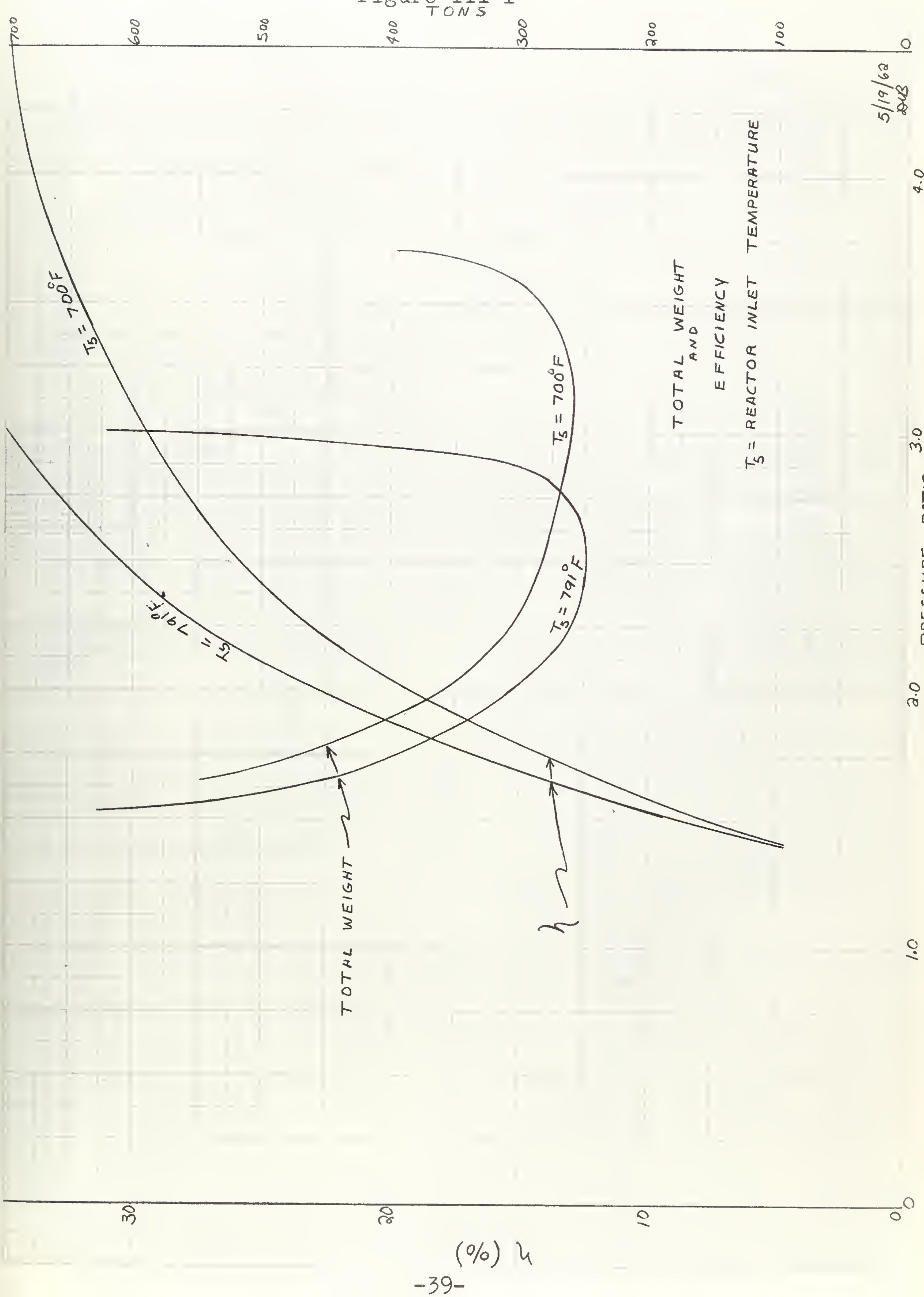

TONS
700
600
500
400
300
200
100
0
5/19/62
DLB
TOTAL WEIGHT AND EFFICIENCY
T_5 = REACTOR INLET TEMPERATURE
$T_5 = 700^\circ F$
$T_5 = 700^\circ F$
$T_5 = 791^\circ F$
$T_5 = 791^\circ F$
TOTAL WEIGHT
η
4.0
3.0
2.0
1.0
0
PRESSURE RATIO
30
20
10
0
η (%)

Figure III-2

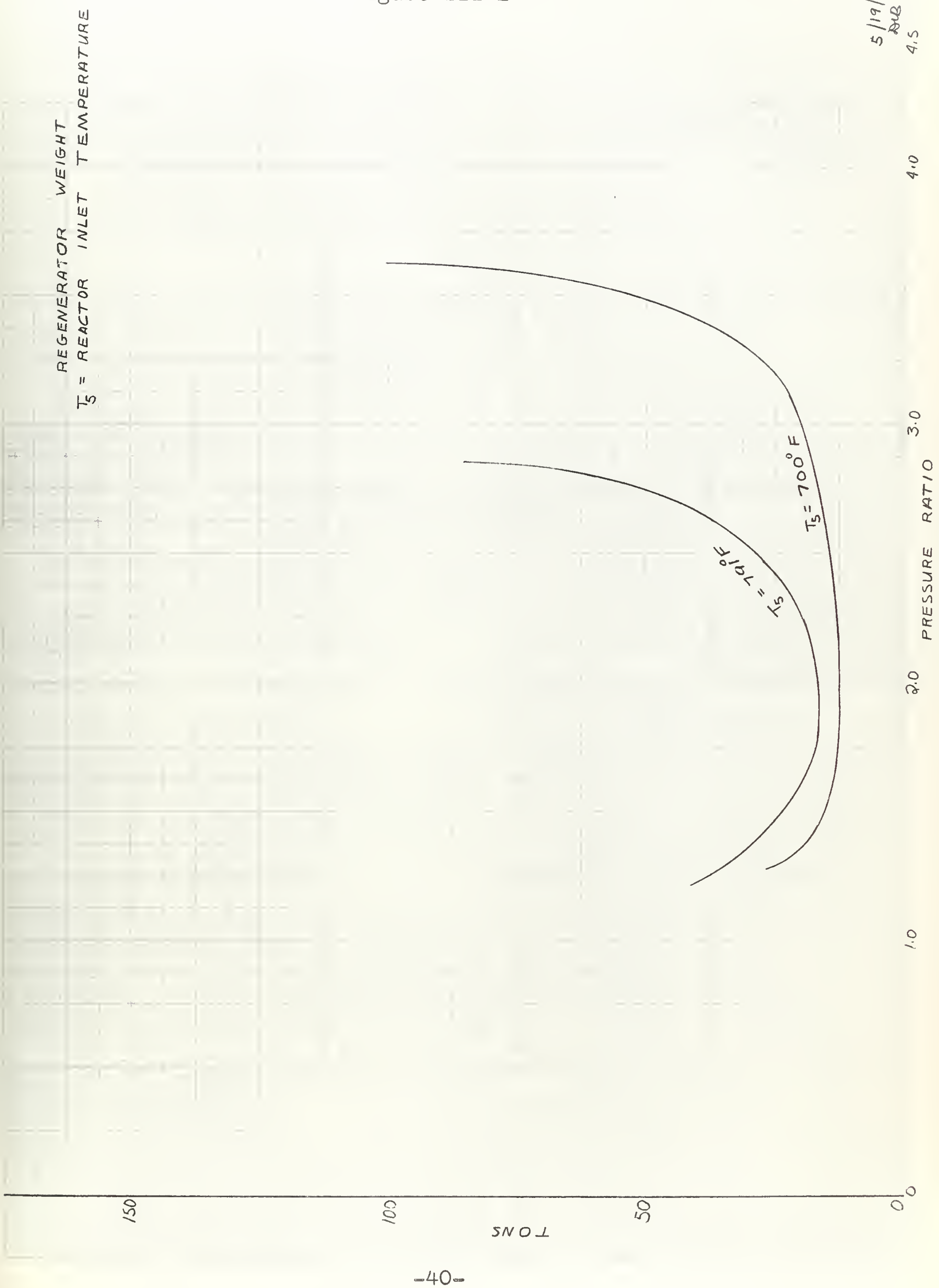
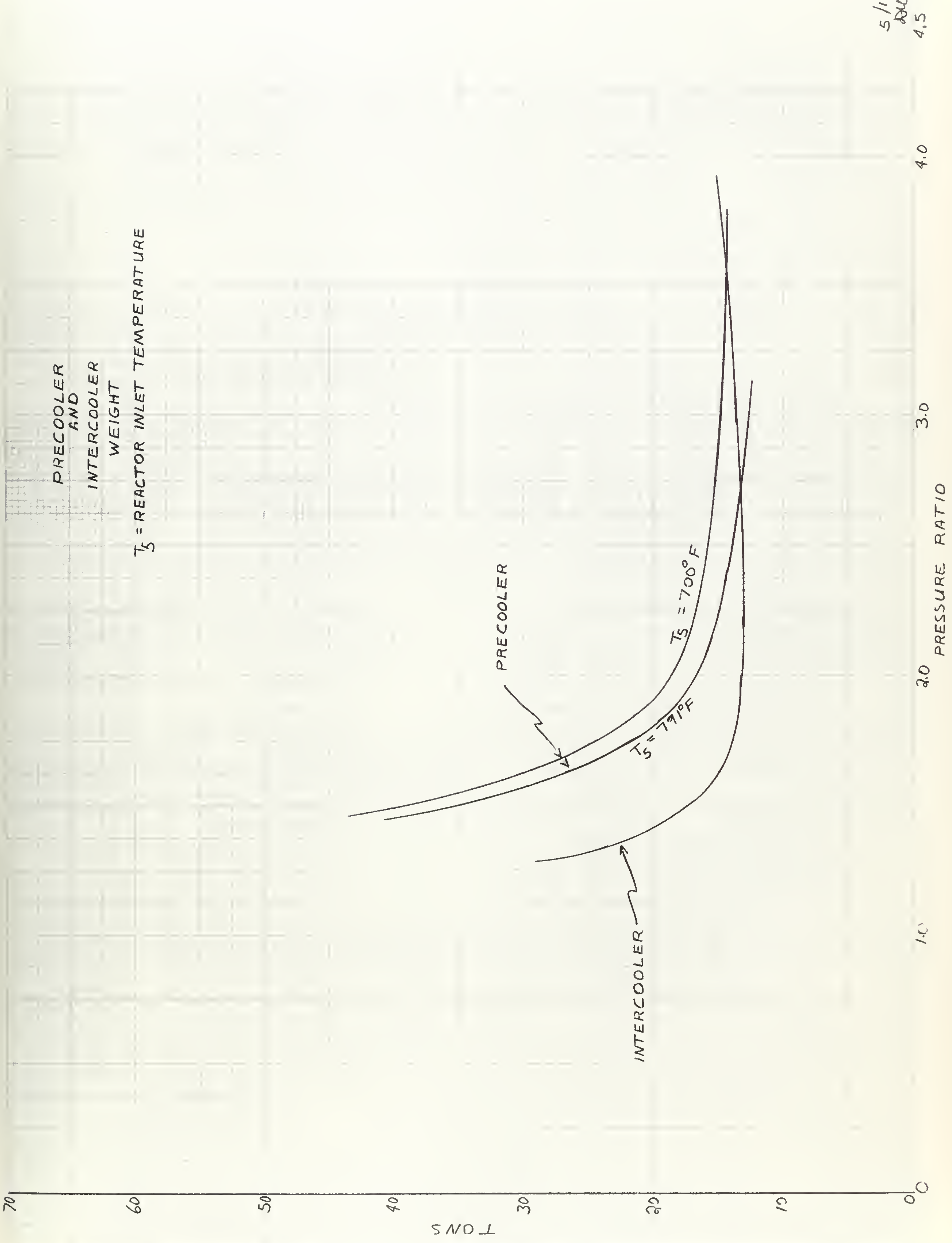

REGENERATOR WEIGHT
T_5 = REACTOR INLET TEMPERATURE
T_5 = 700°F
PRESSURE RATIO
TONS
0
1.0
2.0
3.0
4.0
4.5
0
50
100
150
5/19/62

Figure III-3



5/19/62
RWB
PRECOOLER AND INTERCOOLER WEIGHT
T_5 = REACTOR INLET TEMPERATURE
PRECOOLER
T_5 = 700°F
T_5 = 791°F
INTERCOOLER
PRESSURE RATIO
0
1.0
2.0
3.0
4.0
4.5
TONS
0
10
20
30
40
50
60
70

Figure III-4

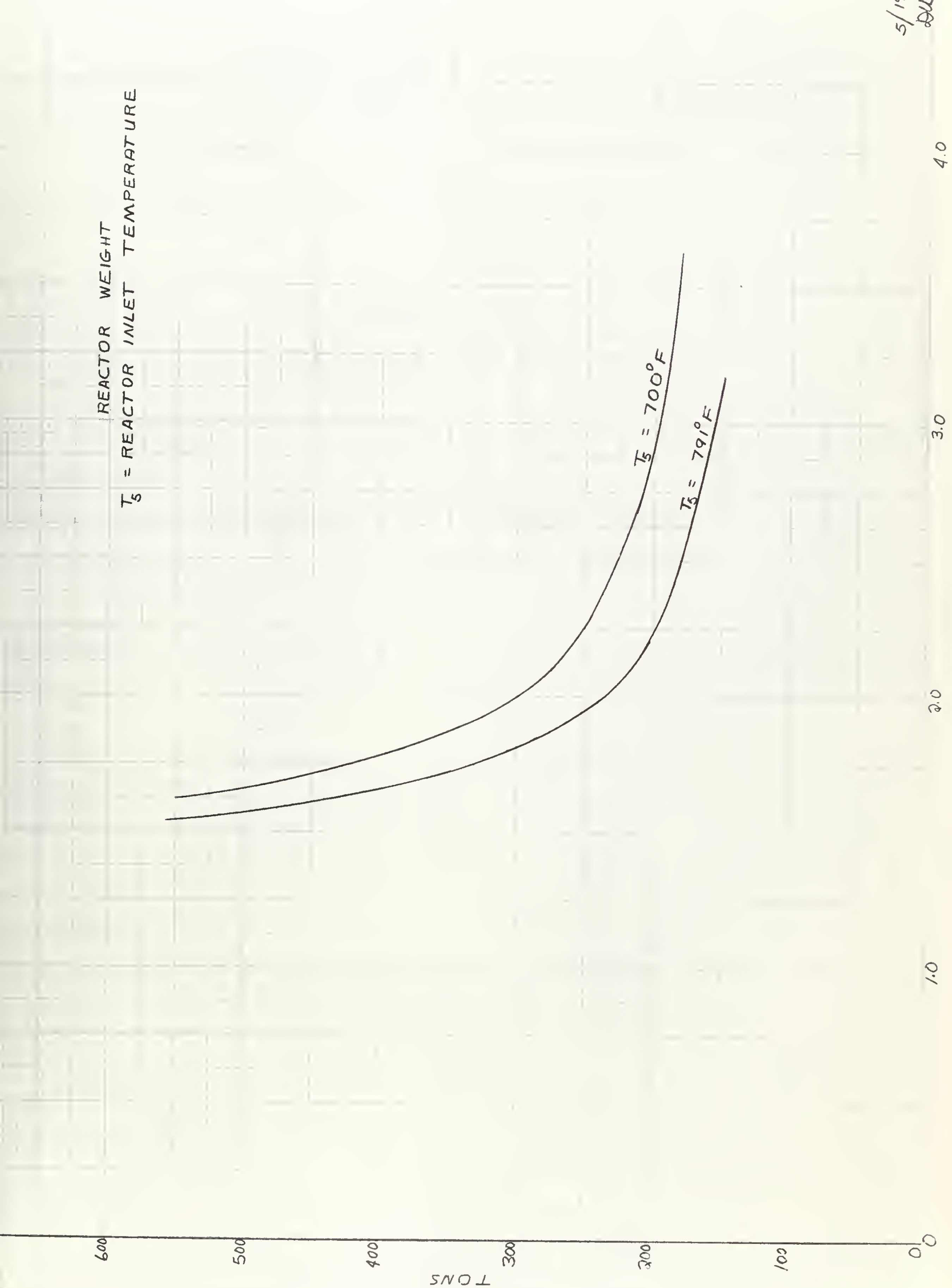

REACTOR WEIGHT
T_5 = REACTOR INLET TEMPERATURE
T_5 = 700°F
T_5 = 791°F
TONS
600
500
400
300
200
100
0
1.0
2.0
3.0
4.0
5/19/62

Figure III-5

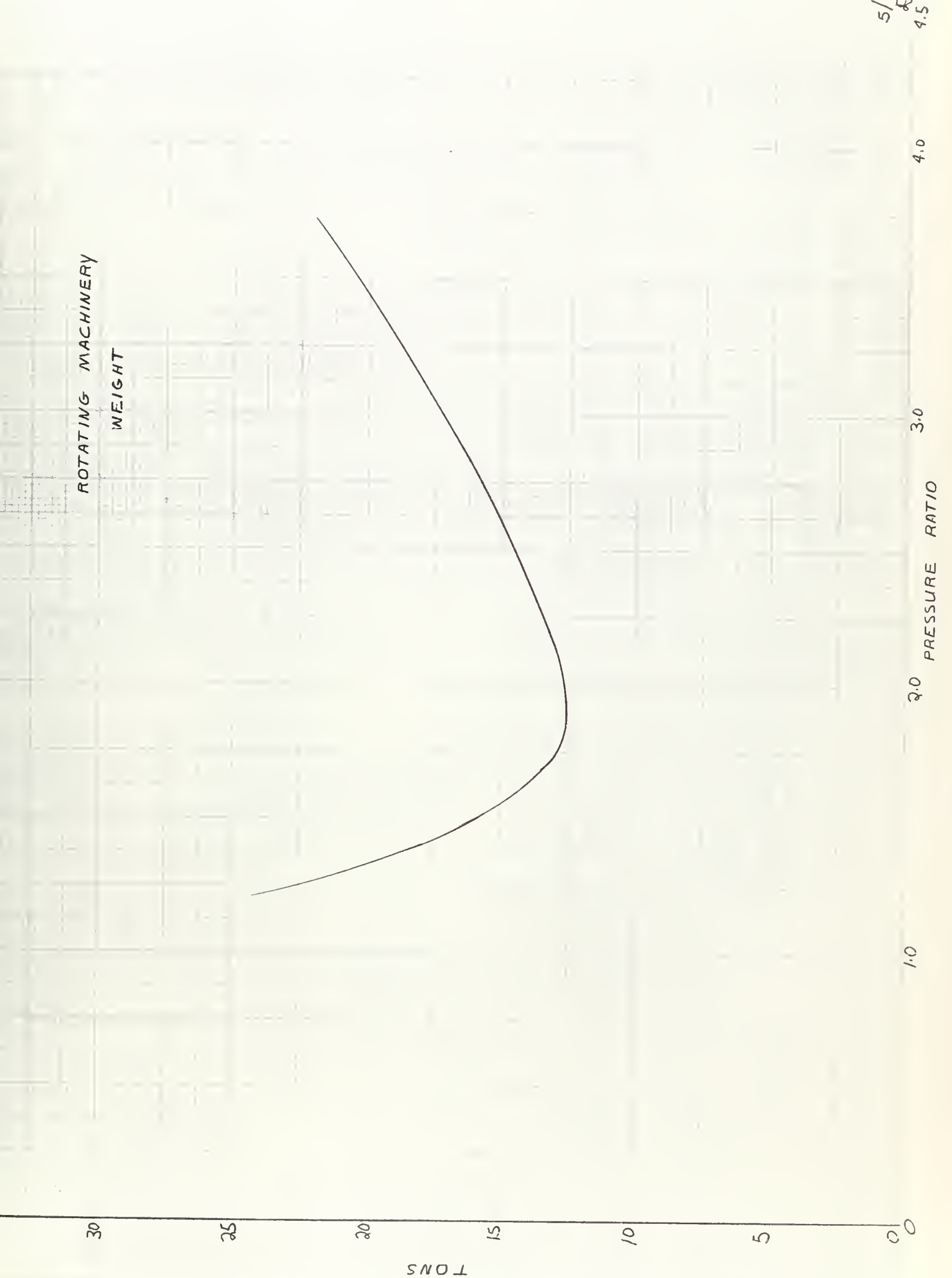

ROTATING MACHINERY WEIGHT
PRESSURE RATIO
TONS
0
1.0
2.0
3.0
4.0
4.5
5
10
15
20
25
30

Figure III-6

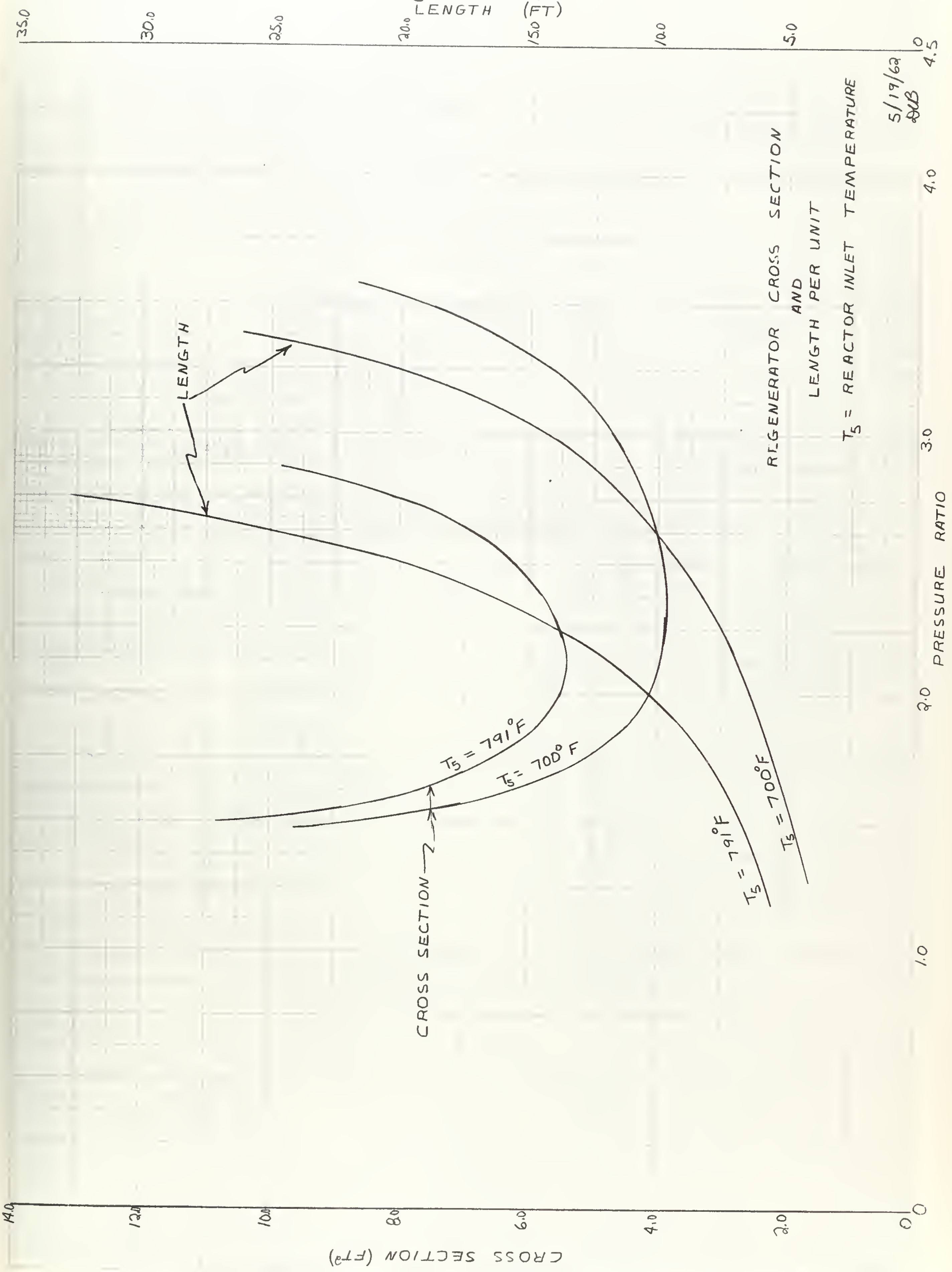

LENGTH (FT)
35.0
30.0
25.0
20.0
15.0
10.0
5.0
0
4.5
4.0
3.0
2.0
1.0
0
REGENERATOR CROSS SECTION
AND
LENGTH PER UNIT
T_5 = REACTOR INLET TEMPERATURE
5/19/62
LENGTH
CROSS SECTION
$T_5 = 791°F$
$T_5 = 700°F$
PRESSURE RATIO
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0
CROSS SECTION (FT²)

Figure III-7

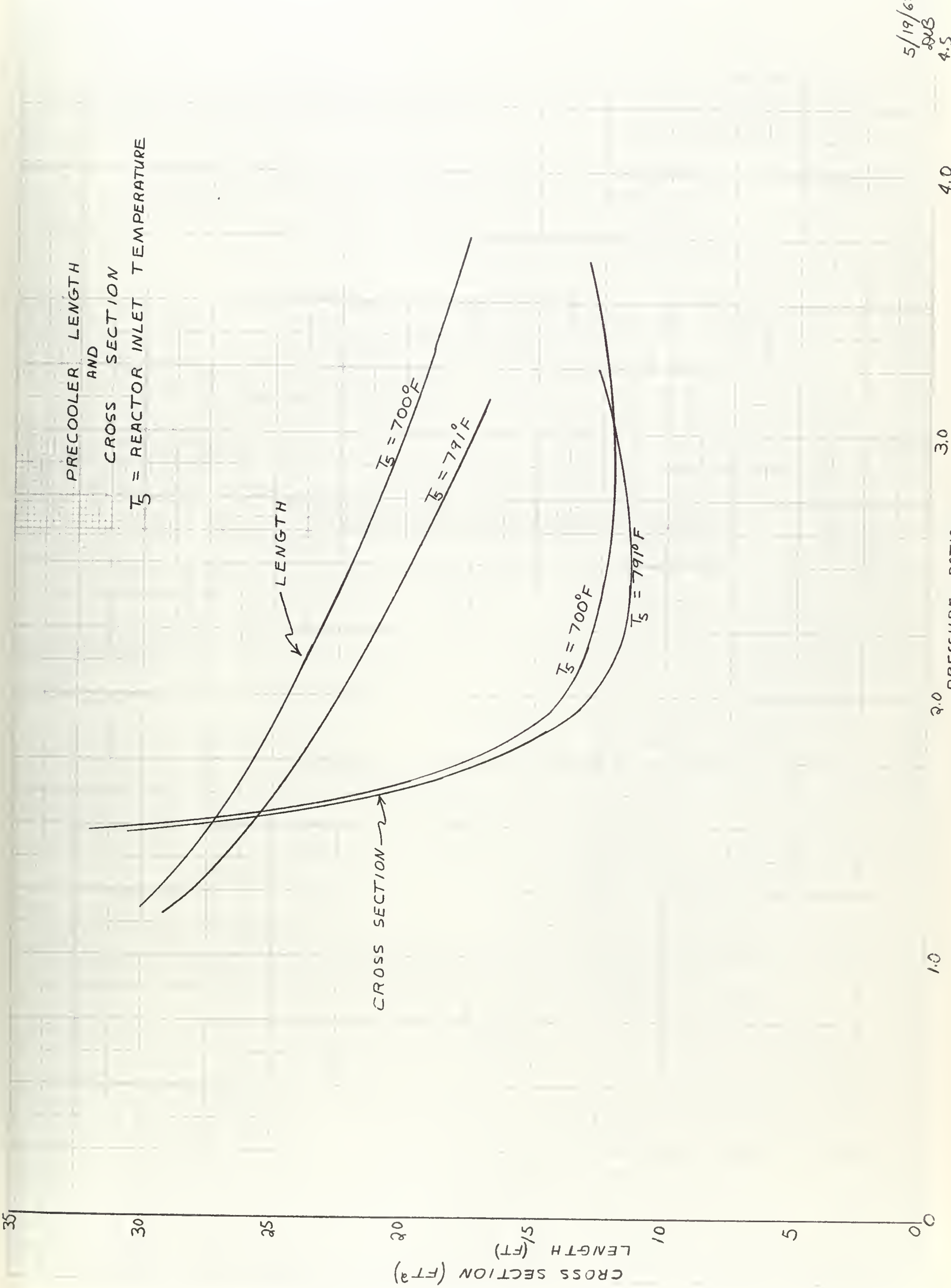

PRECOOLER LENGTH
AND
CROSS SECTION
T_5 = REACTOR INLET TEMPERATURE
LENGTH
CROSS SECTION
$T_5 = 700°F$
$T_5 = 791°F$
$T_5 = 700°F$
$T_5 = 791°F$
PRESSURE RATIO
0
1.0
2.0
3.0
4.0
0
5
10
15
20
25
30
35
LENGTH (FT)
CROSS SECTION (FT²)
5/19/62
4.5

Figure III-8

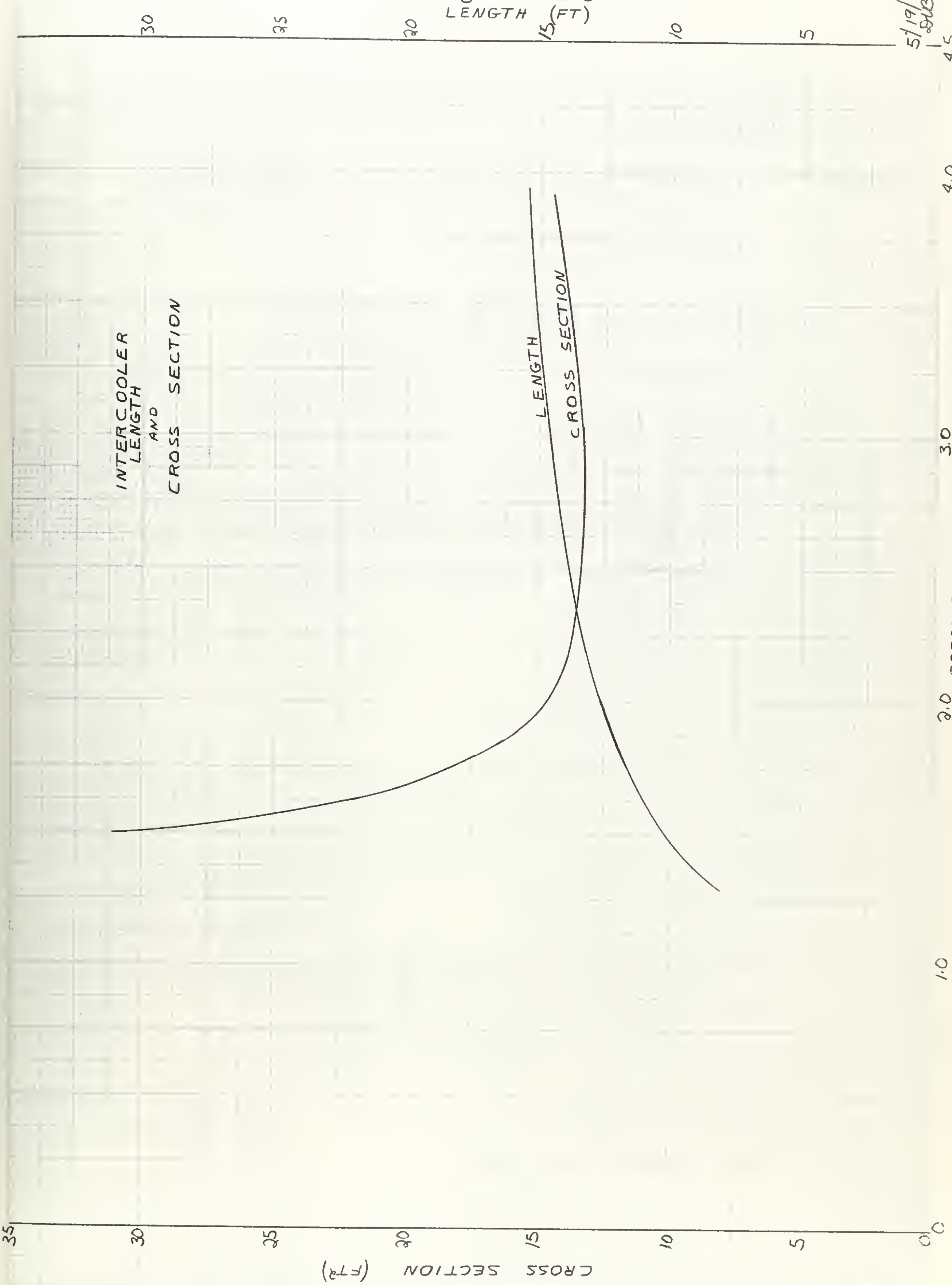

LENGTH (FT)
30
25
20
15
10
5
5/19/62
4.5
4.0
3.0
2.0
1.0
0
PRESSURE RATIO
INTERCOOLER
LENGTH
AND
CROSS SECTION
LENGTH
CROSS SECTION
35
30
25
20
15
10
5
0
CROSS SECTION (FT²)

Figure III-9

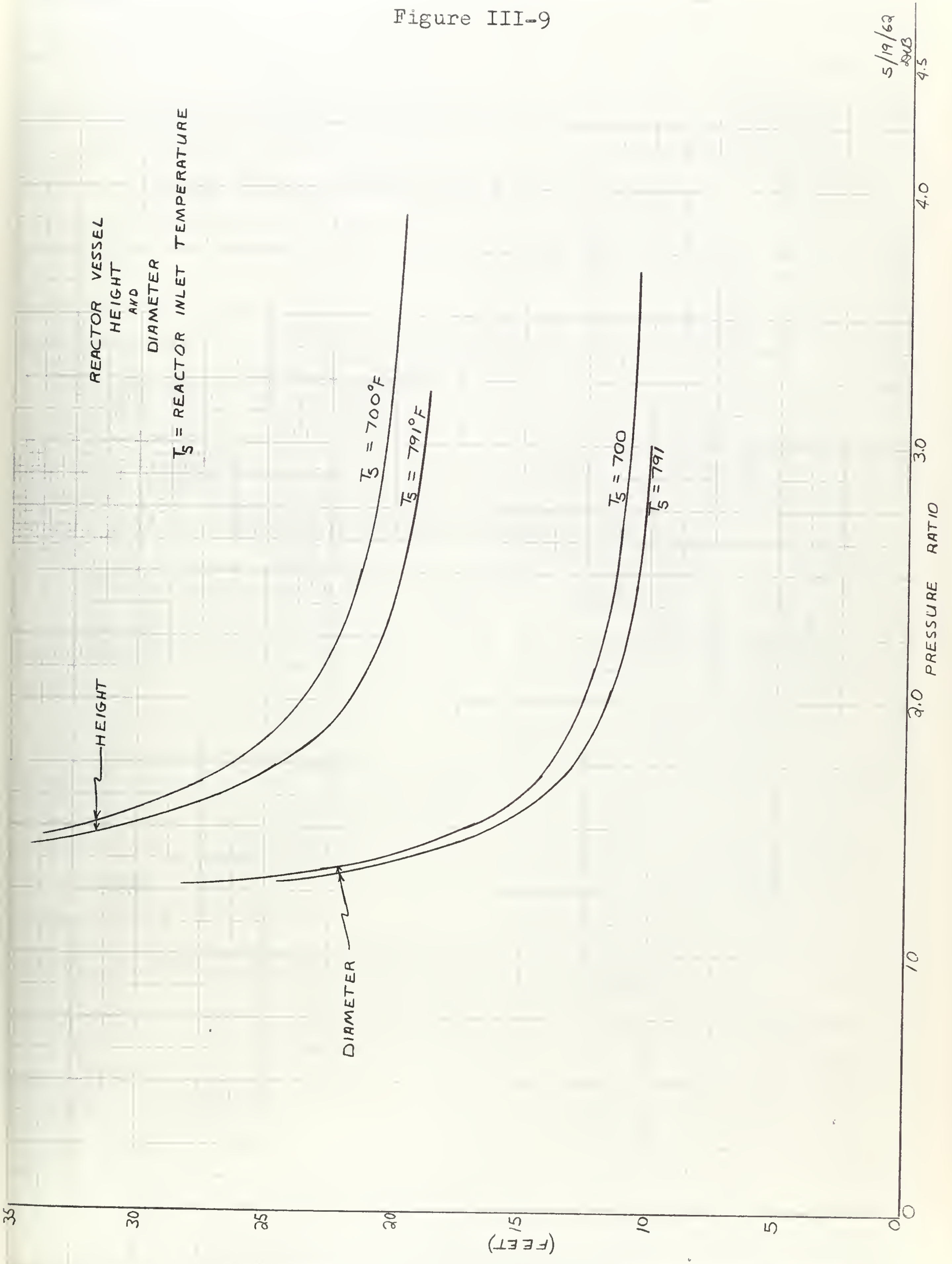

REACTOR VESSEL
HEIGHT
AND
DIAMETER
T_5 = REACTOR INLET TEMPERATURE
HEIGHT
DIAMETER
T_5 = 700°F
T_5 = 791°F
T_5 = 700
T_5 = 791
PRESSURE RATIO
(FEET)
0
5
10
15
20
25
30
35
1.0
2.0
3.0
4.0
4.5
5/19/62

Figure III-10

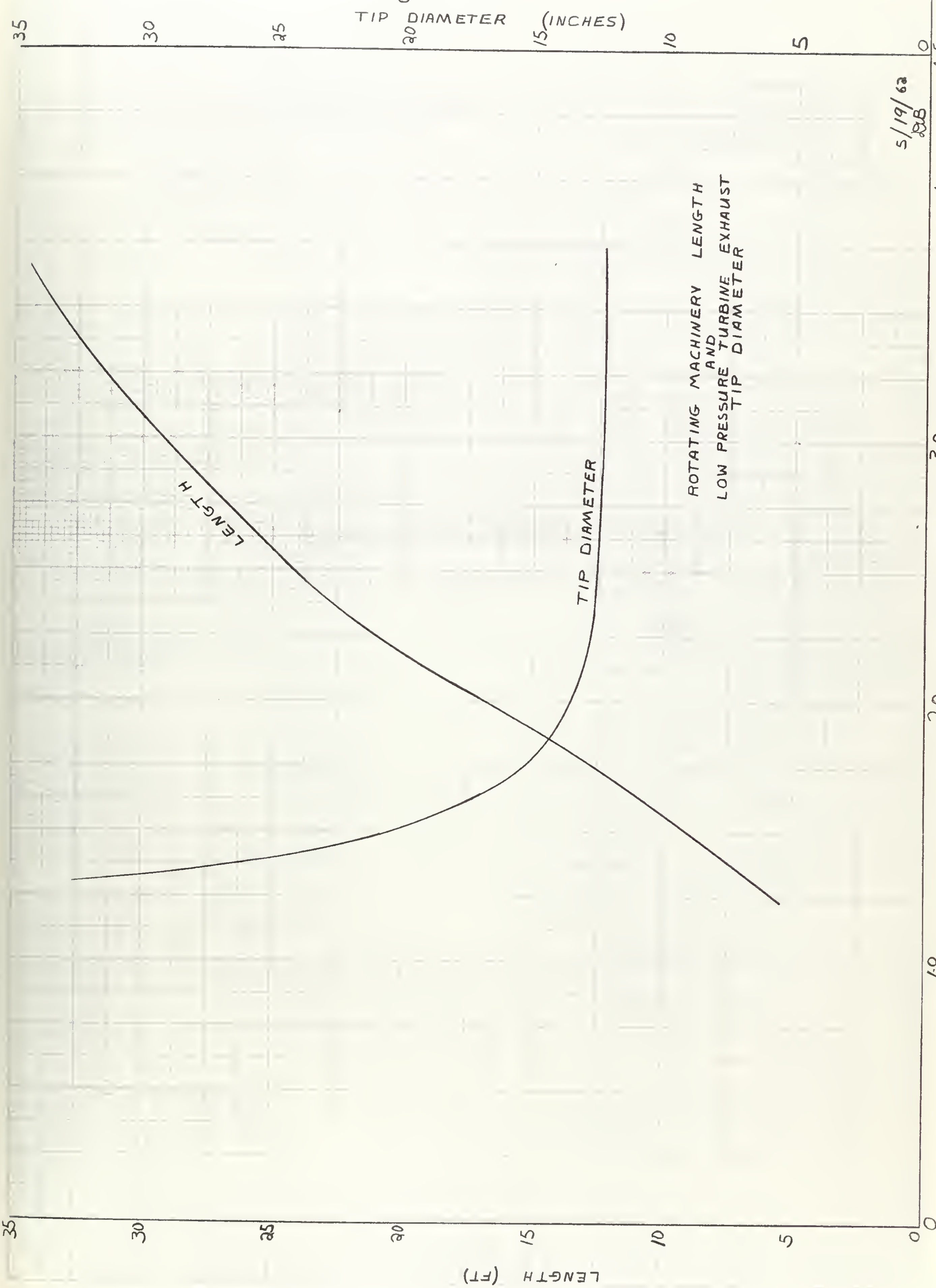
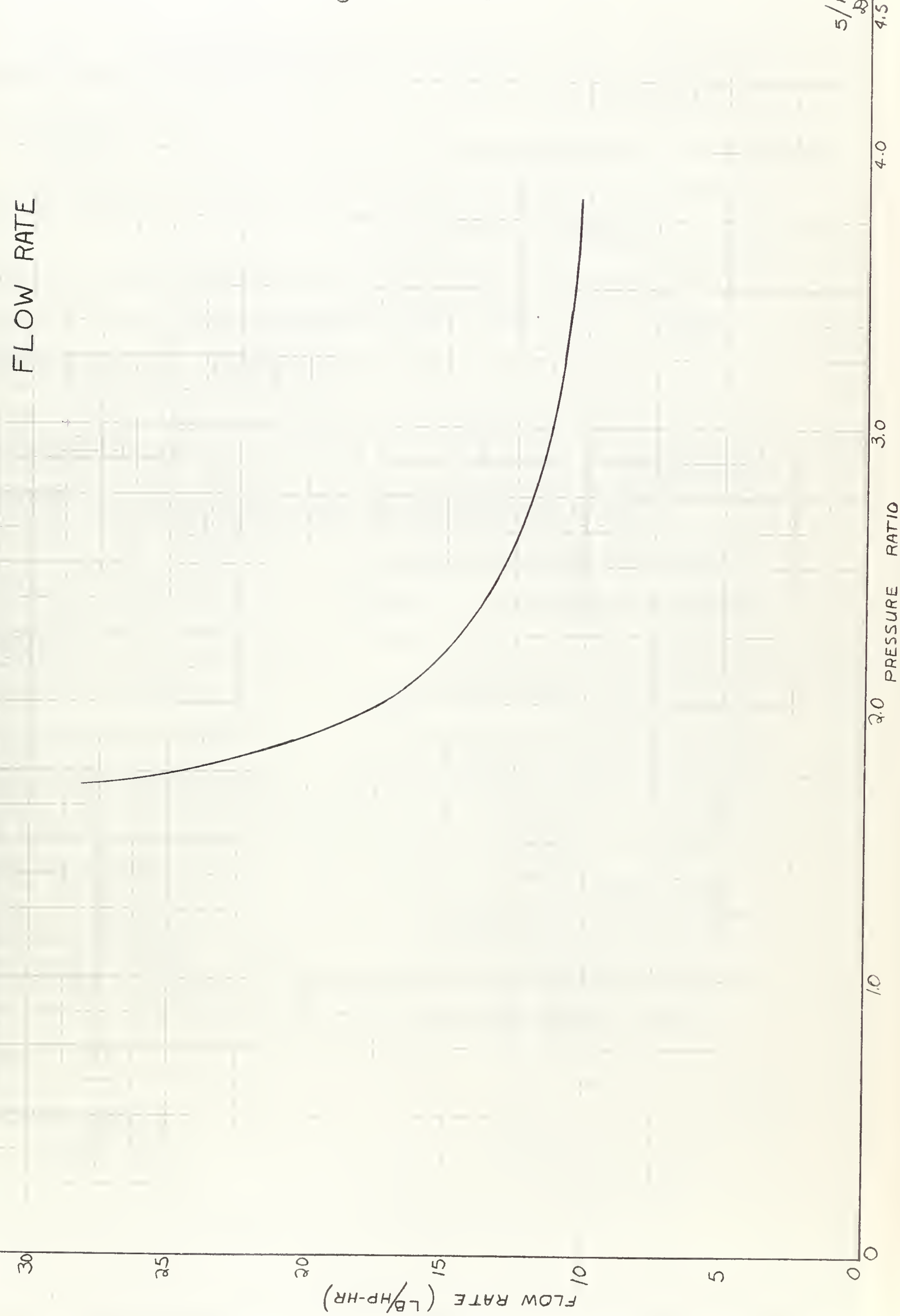


Figure III-11



IV. DISCUSSION OF RESULTS

A. General

The results obtained in this thesis are based on the Maritime Gas-Cooled Reactor (MGCR) plant and are applicable only to a plant of this type. Accuracy of results are dependent on the accuracy with which system constants were evaluated.

Hand calculations were performed to solve the developed equations for a range of operating conditions. Results are presented graphically in Section III.

For all components, the sharp increase in weight or cross sectional area indicated by the curves at low pressure ratios is a result of the large flow rates required at these ratios. The pressure ratios which give a mathematical regenerator effectiveness of 100 per cent are approximately 3.1 and 3.8 respectively for the high and low reactor inlet temperatures. A higher pressure ratio than 3.1 for the cycle with $T_5 = 791^{o}F$ would present an impossible plant since a regenerator effectiveness greater than 100 % is required. A similar statement applies to the cycle with the lower reactor inlet temperature.

B. Total Weight

Total weight and efficiency are plotted against pressure ratio for two different reactor inlet

temperatures in Figure III-1. Minimum weight occurs at a pressure ratio of about 2.5 and gives an efficiency equal to 30.5 % for $T_5 = 791^oF$. For a reactor inlet temperature of 700^oF a slightly higher minimum weight occurs at a pressure ratio equal to 3.15 with efficiency again equal to 30.5 %. For pressure ratios greater than that for optimum weight, the weight rises sharply to infinity due to the regenerator weight. This is because of the large increase in regenerator heat transfer area required as turbine exhaust temperature approaches reactor inlet temperature. The MGCR proposed plant is designed for a pressure ratio of 2.6, a value slightly above that for optimum weight, but giving an efficiency of 32 %.

Figure III-1 indicates that a greater gain in efficiency is possible with small changes in pressure ratio above that ratio which gives optimum weight. Although lowering the reactor inlet temperature gives a flatter optimum weight, overall efficiency is significantly reduced since more heat must be put into the cycle for the same reactor outlet temperature. In addition, the optimum weight for $T_5 = 700^oF$ is larger than that for a higher inlet temperature because of the increased reactor weight. This greater weight is caused by the increase in temperature rise associated with a low inlet temperature as indicated by equation (2-55). The lower inlet

temperature would also tend to add to the complexity of reactor construction.

C. Regenerator

Regenerator weight reaches an optimum at a lower pressure ratio than that for total weight, and rapidly approaches infinity as η_x goes to 100 %. From Figures III-1 and III-2, it is noted that the regenerator is the controlling element in total weight at high pressure ratios. The designer desires a high pressure ratio in order to achieve more efficiency, but without the large increase in weight that must be accepted as indicated by these figures.

A 100 % regenerator effectiveness can never be realized, and in reality the designer must be willing to pay heavily in terms of weight and space for a regenerator effectiveness as low as 90 %. When one considers the weight and efficiency curve for $T_5 = 791^{\circ}F$, it is worthy to note that efficiency can be increased 5 points by going from a pressure ratio of 2.5 to 3.0. However, for this increase in efficiency, total weight is more than doubled. Undoubtedly this sizeable gain in efficiency could not be realized for such a weight penalty.

A possible solution in obtaining part of this increase in efficiency with a less drastic increase in weight might be to use an additional piece of heat

exchanger equipment between the regenerator and reactor inlet. A small quantity of high temperature working fluid could be taken from the reactor. This extraction might occur after the fluid has passed up the outside of the reflector in cooling the thermal shield and vessel wall and before it flows along the surface of the fuel rod assemblies. Another extraction point for the small quantity of high temperature fluid might be in the LPT after partial expansion or at the exhaust of the HPT. This extracted or "bled" fluid would then be recirculated to the additional heat exchanger to further raise the temperature of the bulk of the fluid coming from the regenerator. A considerable portion of the original regenerator duty would then be taken over by the additional heat exchanger (pre-heater). The resulting lower regenerator effectiveness would lower regenerator weight. The heat transmission in this proposed preheater would be very high since both sides of the additional heat exchanger will be near the top pressure in the cycle. With the high rate of heat transmission possible under these conditions, it may be possible to keep the size of the pre-heater quite small. Although this small quantity of "bled" fluid would again enter the reactor at a higher temperature than the bulk of the gas, the thermal inertia of the reactor and the large quantity of fluid should prevent any harmful cyclic

thermal stresses. In order to produce the same power, a larger flow rate of fluid is required with the additional heat exchanger. In addition, a small pump (compressor) would be required to raise the pressure of the "bled" gas to reactor inlet pressure. A study of such a proposal would indicate if the reduction in regenerator weight, caused by a lower regenerator effectiveness, is more than the weight increase caused by the additional heat exchanger (with associated equipment) and the greater weight of other components associated with the slightly larger flow rate. Plant complexity with this system may be such that would not warrant its use.

Figure III-6 indicates the variation of regenerator length from zero to infinity as pressure ratio increases and more heat transfer area is required. The curves indicating length in this figure are for each unit of the 3 unit regenerator component.

D. <u>Precooler</u>

Figure III-3 shows for a pressure ratio of 2 and larger that precooler weight has a slight negative slope. This is to be expected since there is less and less heat transfer area required as the turbine exhaust temperature is lowered with increasing pressure ratio. For the lower reactor inlet temperature, the precooler weight increases. This effect is due to the increased

heat transfer area required of the precooler as regenerator effectiveness decreases.

Precooler length rises with a decreasing pressure ratio as shown in Figure III-7. This would be necessary since there is little temperature drop across the turbines and the fluid must be essentially cooled from the reactor outlet temperature to the fixed low temperature of the cycle. A significant increase in length is required for $T_5 = 700^{\circ}F$ since the precooler must perform more heat transfer as η_x decreases.

E. Intercooler

The intercooler weight, as shown by Figure III-3, increases slightly at higher pressure ratios. As the LPC discharge temperature is raised with pressure ratio, more heat transfer area is required to reduce this temperature to the fixed low temperature of the cycle. There is no change in intercooler heat transfer area (weight) with variation in reactor inlet temperature. The intercooler must always cool the gas from the LPC discharge temperature to the cycle's bottom temperature.

Figure III-8 indicates that intercooler length increases with pressure ratio. For low ratios, little heat transfer area is required to bring the gas back to the cycle's minimum fluid temperature.

F. Reactor

Reactor weight increases sharply at low pressure

ratios since a bulky heat generating unit is required to raise the temperature of the large flow rates. Since reactor weight is directly proportional to flow rate, weight will decrease with less flow. It is also worthy of note that for a drop of 91°F in reactor inlet temperature, a weight increase of approximately 35 tons results at a pressure ratio of 3.0. This is to be expected since reactor weight as given by (2-55) is proportional to temperature rise to the 1.4 power. Reactor vessel diameter and height curves have the same form as the reactor weight curves. The lower inlet temperature requires larger reactor dimensions since equation (2-52) shows that radius cubed is proportional to the temperature rise in the reactor to the 1.4 power.

G. Rotating Machinery

As indicated by Figure III-5, rotating machinery has a minimum weight for a pressure ratio of slightly less than 2.0. At low pressure ratios, the large annulus area required causes an increase in the weight. At higher pressure ratios, the weight increase is not as sharp since length of turbo machinery has less effect on weight than the diameter or annulus area.

Rotating machinery length being a strong function of the number of stages also increases sharply with pressure ratio. Few stages are required at low ratios and many stages are needed at higher pressure

ratios.

Tip diameter for only the low pressure turbine exhaust is indicated in Figure III-10 since its dimensions will be larger than that of the other 3 rotating machines.

Although the rotating machinery weight is significant, the length of the unit is likely to have a greater influence on design of the plant. A machine of great length would necessitate a larger containment vessel with a corresponding increase in shielding weight. An in-line arrangement of compressors and turbines would probably result in lower total rotating machinery weight and fewer shaft sealing, bearing and support problems than an arrangement where the low pressure turbine is in a separate casing. The latter arrangement would permit a shorter overall length by placing the LPT beside the HPT and compressor unit. This arrangement would have a slight detrimental effect on efficiency because of losses in piping between the HPT and LPT, but may be necessary if containment space is limited.

V. CONCLUSIONS

An analysis of the results supports the following conclusions:

1. Optimum weight for the reference plant occurs at a value of pressure ratio that is lower than the value that gives best efficiency.

Efficiency at optimum weight (pressure ratio of 2.5) is 30.5 %. This compares with a maximum possible efficiency of 35.5 % at a pressure ratio of 3.1 (corresponding to a regenerator effectiveness of 100 %) for a reactor inlet temperature of 791°F.

For a reactor inlet temperature of 700°F, the efficiency at optimum weight (pressure ratio of 3.15) is 30.5 %. The maximum possible efficiency is 33.3 % occurring at a pressure ratio of approximately 3.8 (corresponding to a regenerator effectiveness of 100 %).

2. With a plant of this type, the designer must be willing to accept a heavy weight penalty for slight efficiency gains as pressure ratio is increased above the optimum weight value to the ratio that corresponds to a regenerator effectiveness of 100 %. Table V-A indicates the changes in efficiency and total weight for small increments of pressure ratio above that for optimum weight.

Table V-A

Changes in Efficiency and Total Weight vs Pressure Ratio

Pressure ratio increment	$\Delta\eta$ (%)	ΔW (tons)
Reactor inlet temperature = 791°F		
2.5 - 2.6	1.0	5
2.6 - 2.7	1.0	7
2.7 - 2.8	0.8	15
2.8 - 2.9	0.8	55
2.9 - 3.0	0.9	290
Reactor inlet temperature = 700°F		
3.15 - 3.25	.5	4
3.25 - 3.35	.4	5
3.35 - 3.45	.4	10
3.45 - 3.55	.5	16
3.55 - 3.65	.4	40
3.65 - 3.75	.5	165

3. A lower reactor inlet temperature gives a flatter but higher optimum weight. A significant rise in reactor weight results from a lower reactor inlet temperature.

4. The regenerator is the controlling component in this cycle. As heat transfer requirements go up (regenerator effectiveness approaches 100 %), regenerator weight rises sharply and limits the gain in efficiency

that is otherwise possible with a larger pressure ratio.

5. The precooler and intercooler weights and dimensions experience a minimum over a wide range of possible operating pressure ratios and do not strongly influence the total weight.

6. Length restrictions imposed by available space are much more likely to influence rotating machinery design than the weight of this component.

VI. RECOMMENDATIONS

The following recommendations are made:

1. Since most of the developed equations show that weight is inversely proportional to pressure (to a power), the tendency would be to increase pressure level to reduce weight and plant size. However, maximum pressure limitations would be set from the high temperature, pressure effects on materials. For a more complete analysis an investigation should be made to determine the effect of pressure on the reactor, piping and regenerator weight.

2. Equations should be refined to give greater accuracy. Accuracy can be improved as plants of this type are designed and built. A survey of these plants for component weight and size data will contribute to a more precise evaluation of system constants. Such a survey would provide information for proper evaluation of the piping and accumulator constants.

3. Shielding accounts for a major portion of total plant weight, with the secondary shielding weight being largely a function of size and arrangement of components within the containment vessel. A thorough study of size and arrangement of components is necessary to ensure a minimum shielding weight. Reference [9] gives a suitable method for the preliminary evaluation of shielding weight.

4. The regenerator weight and size dominates that of all other components by several orders of magnitude as regenerator effectiveness approaches 100 %. An investigation as to the feasibility of recirculating a small quantity of fluid from the reactor after it has cooled the thermal shield or from the turbine (after partial expansion) through an additional heat exchanger (pre-heater), located between the regenerator and reactor inlet, is necessary. Such a study would indicate if the reduction in regenerator weight, caused by a lower regenerator effectiveness, is more than the weight increase caused by the additional heat exchanger and the greater weight of other components associated with the slightly larger flow rates.

VII. APPENDIX

APPENDIX A

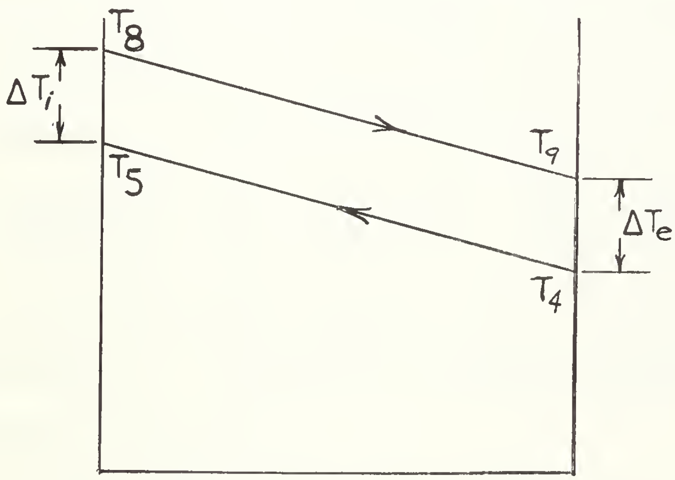
HEAT EXCHANGER EQUATIONS DEVELOPMENT

I. General

Development of equations for the heat exchanger cross sectional and heat transfer area variation are based on reference [17].

Pressure drop of the gas due to entry and exit effect in the exchangers are considered negligible.

II. Regenerator



Z - distance along regenerator

Figure A-1

Regenerator Temperature-Length Diagram

The rate of heat transfer from the hot fluid to the cold fluid for equal flow rates and heat capacities on each side may be written in the form:

$$Q = wC_p \ (T_5 - T_4) \tag{A-1}$$

$$Q = wC_p \ (T_8 - T_9) \tag{A-2}$$

$$Q = UA_s \triangle T_m \tag{A-3}$$

where $\Delta T_m = \Delta T_i = \Delta T_e$ is the log mean temperature difference across the counter flow regenerator shown in Figure A-1.

Regenerator effectiveness is defined as:

$$\eta_x = \frac{T_5 - T_4}{T_8 - T_4} = \frac{T_8 - T_9}{T_8 - T_4} \qquad \text{(A-4)}$$

Combining equations (A-1), (A-2), (A-3) and (A-4):

$$A_s U = \frac{Q}{\Delta T_m} = w C_p \left(\frac{\eta_x}{1 - \eta_x} \right) \qquad \text{(A-5)}$$

The overall coefficient of heat transfer is written as:

$$\frac{1}{UA_s} = \frac{1}{H_h A_{sh}} + \frac{\ln(d_o/d_i)}{k 2 \pi L} + \frac{1}{H_c A_{sc}} \qquad \text{(A-6)}$$

For a gas-to-gas heat exchanger, the thermal resistance of the tube wall may be neglected in comparison with the film resistances and (A-6) may be written:

$$\frac{1}{U} = \frac{1}{H_h} \frac{A_{sc}}{A_{sh}} + \frac{1}{H_c} = \frac{1}{H_c} \left(1 + \frac{H_c}{H_h} \frac{d_i}{d_o} \right) \qquad \text{(A-7)}$$

where $A_s = A_{sc}$, the heat transfer area inside the tube.

Reference [5] expresses the heat transfer coefficient of a turbulent fluid as:

$$H = .023 \frac{k_f}{d_e} (Re)^{.8} (Pr)^{.4} \qquad \text{(A-8)}$$

The hydraulic diameter outside of the tubes (hot side) becomes

$$d_{eh} = d_o q \qquad \text{(A-9)}$$

where q for triangular tube spacing is:

$$q = \frac{2(3)^{.5}}{\pi} \left(\frac{p}{d_o} \right)^2 - 1 \qquad \text{(A-9A)}$$

The equivalent outside flow diameter per tube can be written as:

$$d_x = d_o (q)^{.5} \qquad \text{(A-10)}$$

For the cold side, Reynolds number can be written:

$$Re)_c = \frac{4w\ C_p}{\pi d_i\ N\mu_c C_p} \qquad \text{(A-11)}$$

and for the hot side:

$$Re)_h = \frac{4w\ C_p}{\pi d_o N\mu_h C_p} \qquad \text{(A-12)}$$

Substituting (A-11) into (A-8) and noting that $d_e = d_i$ for the cold side:

$$H_c = \frac{.028\ k_c^{.2}\ (wC_p)^{.8}}{N^{.8}\ d_i^{1.8}\ (Pr)_c^{.4}} \qquad \text{(A-13)}$$

(A-8) becomes for the hot side:

$$H_h = \frac{.028\ k_h^{.2}\ (wC_p)^{.8}}{N^{.8}\ d_o^{1.8}\ q(Pr)_h^{.4}} \qquad \text{(A-14)}$$

Combining equations (A-7), (A-13) and (A-14):

$$\frac{1}{U} = \frac{N^{.8} d_i^{1.8} (Pr)_c^{.4}}{.028\ (wC_p)^{.8} k_c^{.2}} \left(1 + \ell\, q\left(\frac{d_o}{d_i}\right)^{.8}\right) \qquad \text{(A-15)}$$

where $\ell = \frac{k_c}{k_h}^{.2} \frac{(Pr)_h^{.4}}{(Pr)_c^{.4}} \approx 1$ for the temperature range considered. The overall heat transfer coefficient then becomes:

$$U = \frac{.028 (wC_p)^{.8} k_c^{.2}}{N^{.8} d_i^{1.8} (Pr)_c^{.4}} \left(\frac{1}{1 + q\left(\frac{d_o}{d_i}\right)^{.8}}\right) \qquad \text{(A-16)}$$

The pressure drop of a gas flowing through a tube with combined friction and heat transfer can be expressed for low mach numbers [7] as:

$$\frac{dP_o}{P_o} = -\frac{KM^2}{2}\left(4f\,\frac{L}{d_e}\right)\left(1 + \frac{1}{2}\,\frac{T_w - T_o}{T_o}\right) d\left(\frac{x}{L}\right) \qquad \text{(A-17)}$$

where f is the Fanning friction factor defined as $\dfrac{\text{shear stress}}{\rho V^2/2g_o}$. In the regenerator and coolers, the term $\frac{1}{2}\left(\frac{T_w - T_o}{T_o}\right)$ is negligible compared to 1 and is dropped. With the low mach number likely to be encountered in the ducting and heat exchanger equipment, static pressures and temperatures can be substituted for stagnation pressures and temperatures with little loss of accuracy. With these simplifications, (A-17) can be integrated to give for the cold side:

$$\ell n \; \frac{P_5}{P_4} = -K\,\frac{M_c^2}{2}\;4\;f_c\;\frac{L}{d_i} \qquad \text{(A-18)}$$

and for the hot side:

$$\ell n \; \frac{P_9}{P_8} = -K\,\frac{M_h^2}{2}\;4\;f_h\;\frac{L}{d_o q} \qquad \text{(A-19)}$$

From equation (2-4), the pressure drop factor (Y_{RG}) contributed by the regenerator is:

$$Y_{RG} = \left(\frac{P_5}{P_4}\,\frac{P_9}{P_8}\right)^V \qquad \text{(A-20)}$$

Take antilog of (A-18) and (A-19) and combine with (A-20) to obtain:

$$Y_{RG} = \left[\exp\left(-\frac{K}{2}\,4L\left(\frac{M_c^2 f_c}{d_i} + \frac{M_h^2 f_h}{d_o q}\right)\right)\right]^V \qquad \text{(A-21)}$$

Y_{RG} is a constant for the working fluid used and for a particular top pressure in the cycle.

After taking the logarithm of (A-21):

$$y_{RG} = \left(\frac{K-1}{2}\right)(4L)\left(\frac{M_c^2 f_c}{d_i} + \frac{M_h^2 f_h}{d_o q}\right) \quad \text{(A-22)}$$

y_{RG} is a constant since Y_{RG}, as noted above, is constant.

The relationship for the Fanning friction factor (f) for turbulent flow can be given as:

$$f = \frac{.046}{(Re)^{.2}} \quad \text{(A-23)}$$

Substituting (A-11) and (A-12) into (A-23):

$$f_c = .044 \left(\frac{d_i N \mu C_p}{wC_p}\right)_c^{.2} \quad \text{(A-24)}$$

$$f_h = .044 \left(\frac{d_o N \mu C_p}{wC_p}\right)_h^{.2} \quad \text{(A-25)}$$

From the mach number definition and the continuity relation, M^2 for the cold and hot side may be written respectively as:

$$M_c^2 = \left(\frac{4J}{\pi}\right)^2 \frac{1}{g_o} (wC_p)^2 \frac{T_c}{RK} \left(\frac{K-1}{K} \frac{1}{P_c N d_i^2}\right)^2 \quad \text{(A-26)}$$

$$M_h^2 = \left(\frac{4J}{\pi}\right)^2 \frac{1}{g_o} (wC_p)^2 \frac{T_h}{RK} \left(\frac{K-1}{K} \frac{1}{P_h N q d_o^2}\right)^2 \quad \text{(A-27)}$$

where $T_c = \dfrac{T_5 + T_4}{2}$ and $T_h = \dfrac{T_8 + T_9}{2}$.

Substituting (A-24), (A-25), (A-26) and (A-27) into (A-22) and simplifying, the following expression is obtained:

$$y_{RG} = C_{RG} \left(\frac{K-1}{K}\right)^3 \frac{L}{R} \frac{(wC_p)^{1.8}}{N^{1.8}} \frac{(\mu C_p)^{.2}}{d_i^{4.8}} \frac{1}{P^2} \left[T_c + r^2 \frac{T_h}{q^3} \left(\frac{d_i}{d_o}\right)^{4.8}\right] \quad \text{(A-28)}$$

where it was assumed $(\mu C_p)_c \approx (\mu C_p)_h$ at the mean temperatures of the cold and hot side, and all constants have been written as C_{RG}. The pressure term in the denominator of (A-28) is the top pressure level in the cycle.

Combining (A-3), (A-5) and (A-16):

$$wC_p \frac{\eta_x}{1-\eta_x} = \frac{.028\ (wC_p)^{.8}\ k_c^{\ .2}}{N^{.8}\ d_i^{\ 1.8} (Pr)_c^{.4}} \left(\frac{1}{1 + q\left(\frac{d_o}{d_i}\right)^{.8}}\right) \pi N d_i L \quad \text{(A-29)}$$

Divide equation (A-28) by (A-29) and solve the resulting expression for Nd_i^2:

$$Nd_i^2 = C_{ND} \frac{wC_p}{R^{.5}} \left(\frac{\eta_x}{1-\eta_x}\right)^{.5} \left(\frac{K-1}{K}\right)^{1.5} (Pr)^{.3} \frac{1}{P} \left[1+q\left(\frac{d_o}{d_i}\right)^{.8}\right]^{.5}$$

$$\left[T_c + r^2 \frac{T_h}{q^3} \left(\frac{d_i}{d_o}\right)^{4.8}\right]^{.5} \quad \text{(A-30)}$$

Equation (A-30) represents the cross sectional area variation of the regenerator in terms of gas properties, tube size and arrangement, regenerator effectiveness, flow rate, top pressure level, cycle pressure ratio, and mean temperatures on the hot and cold side. C_{Nd} represents the constant terms.

The heat transfer area variation (Nd_iL) is obtained as a function of the same properties by solving (A-29) for L/d_i, making the appropriate substitution for $(Nd_i^2)^{.2}$ and multiplying this result by (A-30):

$$Nd_iL = C_{NdL}\, wC_p \left(\frac{\eta_x}{1-\eta_x}\right)^{1.4} \left(\frac{K-1}{K}\right)^{1.2} (Pr)^{.64} \left(\frac{1}{R}\right)^{.4}$$

$$\left(\frac{d_i}{k}\right)^{.2} \frac{1}{P^{.8}} \left[1+q\left(\frac{d_o}{d_i}\right)^{.8}\right]^{1.4} \left[T_c + r^2\ \frac{T_h}{q^3}\left(\frac{d_i}{d_o}\right)^{4.8}\right]^{0.4} \qquad \text{(A-31)}$$

It is noted that equations (A-30) and (A-31) simplify considerably for a given gas, tube size and tube arrangement. The resulting expressions then become functions of flow rate, regenerator effectiveness, top pressure level, pressure ratio and mean temperatures on the hot and cold side. In the last two equations, the gas properties may be evaluated at the mean temperature on the hot or cold side since Pr and the ratio of specific heats can be considered constant over a large range of temperatures for most gases. Also the change in thermal conductivity to the 0.2 power would vary only slightly since k undergoes little change with limited temperature variation.

III. Coolers

Development of the corresponding cross sectional and heat transfer area equations was done using the same analogy as for the regenerator. For the coolers

the heat transfer coefficient was based on the outside of the tubes (gas side). With the approximation that the tube wall and water film thermal resistances are small compared to the gas thermal resistance, the overall heat transfer coefficient was based solely on the gas heat transfer coefficient.

For the precooler, the cross sectional area variation is then given by:

$$Nd_o^2 = C_{Nd}\, w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{.5} \left(\frac{T_{pc}}{R}\right)^{.5} \left(\frac{K-1}{K}\right)^{1.5} \frac{(Pr)^{.3}}{q} \quad \frac{r}{P} \qquad \text{(A-32)}$$

The heat transfer area variation is:

$$Nd_oL = C_{NdL}\, w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{1.4} \left(\frac{T_{pc}}{R}\right)^{.4} \left(\frac{K-1}{K}\right)^{1.2} (Pr)^{.64} \left(\frac{d_o q}{k}\right)^{.2} \left(\frac{r}{P}\right)^{.8} \qquad \text{(A-33)}$$

Similarly for the intercooler, the following results:

$$Nd_o^2 = C_{Nd}\, w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{.5} \left(\frac{T_{ic}}{R}\right)^{.5} \left(\frac{K-1}{K}\right)^{1.5} \frac{(Pr)^{.3}}{q} \quad \frac{r^{.5}}{P} \qquad \text{(A-34)}$$

$$Nd_oL = C_{NdL}\, w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{1.4} \left(\frac{T_{ic}}{R}\right)^{.4} \left(\frac{K-1}{K}\right)^{1.2} (Pr)^{.64} \left(\frac{d_o q}{k}\right)^{.2} \frac{r^{.4}}{P^{.8}} \qquad \text{(A-35)}$$

where R, K, k, and Pr are the familiar gas properties. In the above expressions, the gas properties are evaluated at the mean gas temperature in the respective cooler.

$$T_{pc} = \frac{T_9 + T_1}{2} \quad \text{and} \quad T_{ic} = \frac{T_2 + T_3}{2} = \frac{T_2 + T_1}{2}$$

P and r are again the top pressure level and pressure ratio respectively of the cycle. For the precooler:

$$\Delta T_g = T_9 - T_1$$

and ΔT_m is log mean temperature difference across the precooler.

For the intercooler:

$$\Delta T_g = T_2 - T_3 = T_2 - T_1$$

and ΔT_m is the LMTD across the intercooler.

APPENDIX B

TURBO-MACHINERY EQUATIONS DEVELOPMENT

I. General

The assumptions involved in the development of the rotating machinery equations include:

1. Uniform axial velocity over the length of the component.
2. All machines of free vortex design.
3. Equal work done per turbine or compressor stage.
4. Static pressure and temperature equal to stagnation pressure and temperature.
5. Steady flow.

Development of the rotating machinery equations are in part based on reference [6].

II. Compressors

The root stress in a blade caused by centrifugal force can be expressed in accordance with references [6] and [8] as:

$$\delta = f \frac{\rho}{g_o} \frac{\Omega^2}{2} d^2 \left(\frac{\ell}{d}\right) \tag{B-1}$$

where f is the blade taper factor and ρ the density of blade material. Defining a stress parameter (β) as equal to $\left(\frac{\delta}{f\rho}\right)$, (B-1) is written as:

$$\beta = \frac{\Omega^2}{2g_o} d^2 \left(\frac{\ell}{d}\right) \tag{B-2}$$

Since the annular flow area is:

$$A = \pi d \ell \qquad \text{(B-3)}$$

(B-2) can be written as:

$$\beta = \frac{\Omega^2 A}{2 \pi g_o} = \frac{4 U^2}{2 g_o}\left(\frac{\ell}{d}\right) \qquad \text{(B-4)}$$

where the relation $\Omega = \frac{U}{d/2}$ was used to obtain the term on the right in (B-4). From continuity and the perfect gas relationship:

$$AC_a = \frac{wRT}{P} \qquad \text{(B-5)}$$

Using (B-4), (B-5) and the relation RPM $= \frac{60 \Omega}{2\pi}$, machinery rotational speed can be expressed:

$$RPM = 30 \left[\frac{P(C_{a/U})}{\pi \ wRT}\right]^{.5} \left[\frac{2 \ g_o^3 \ \beta^3}{(\ell/d)}\right]^{.5} \qquad \text{(B-6)}$$

From (B-6) it is seen that RPM may be limited because of stress considerations. For a fluid with low sonic velocity, RPM will be limited by mach number considerations. For RPM designed with mach number considerations, equation (B-6) becomes:

$$RPM = 60\left(\frac{Kg_o}{\pi}\right)^{.5} \frac{U}{\left(Kg_oRT\right)^{.5}} \left(\frac{\ell}{d}\right)^{.5} (PC_a)^{.5} \qquad \text{(B-7)}$$

after substituting (B-4) for β. All remaining turbo machinery relationships will be based on stress considerations. For helium as a working fluid, compressor and turbine design would be influenced by stress considerations rather than mach number limitations.

(B-3) can be written as:

$$A = \pi d^2 \frac{\ell}{d} \qquad \text{(B-8)}$$

and $$d = \left(\frac{A}{\pi \ell/d}\right)^{.5} \tag{B-9}$$

Using (B-4) and (B-5), the pitch diameter is represented by:

$$d = \left[\frac{2\ wRT}{P(C_{a/U})}\right]^{.5} \left[2g_o\beta\ \frac{\ell}{d}\right]^{-.25} \tag{B-10}$$

For the LPC inlet, (B-10) becomes:

$$d_{LPC} = K\left(\frac{wT_1 r}{P}\right)^{.5} \left(\frac{1}{C_{a/U}}\right)^{.5} \left(\beta\ \frac{\ell}{d}\right)^{-.25} \tag{B-11}$$

where P and r are the top pressure level and the pressure ratio respectively of the cycle, and K represents the constant terms.

Substituting (B-11) into (B-8), the inlet annulus area for the LPC is:

$$A_{LPC} = K\ \frac{wT_1 r}{P(C_{a/U})}\ (\beta)^{-.5} \left(\frac{\ell}{d}\right)^{.5} \tag{B-12}$$

For most compressors, β, $\frac{\ell}{d}$, and $C_{a/U}$ can be assigned fixed values over a range of operating conditions. Then pitch diameter and annulus area are dependent on the volumetric rate of flow and cycle pressure ratio.

Using the same procedure, the corresponding equations for the HPC are:

$$d_{HPC} = K\left(\frac{wT_1}{P}\right)^{.5} (r)^{.25} \left(\frac{1}{C_{a/U}}\right)^{.5} \left(\beta\ \frac{\ell}{d}\right)^{-.25} \tag{B-13}$$

$$A_{HPC} = K\ \frac{wT_1}{P}\ (r)^{.5} \left(\frac{1}{C_{a/U}}\right) (\beta)^{-.5} \left(\frac{\ell}{d}\right)^{.5} \tag{B-14}$$

The tip diameter at the inlet end of the LPC can be expressed as :

$$D_t = \left[1 + \left(\frac{\ell}{d}\right)_1\right] d_{LPC} \tag{B-15}$$

with a similar expression for the HPC.

Applying Newton's law in the form of angular momentum, along with the assumptions given in section I of this appendix, the work done per compressor stage is:

$$W_s = \frac{U^2}{g_o} \frac{C_a}{U} (\tan \beta_2 - \tan \beta_1) \qquad \text{(B-16)}$$

where β_2 and β_1 are the angles as shown in Figure B-1.

For the LPC, the work is given by:

$$W_{LPC} = \frac{J\ C_p T_1}{\eta_c} (\tau^{.5} - 1) \qquad \text{(B-17)}$$

Dividing (B-17) by (B-16) the number of stages is:

$$N_{LPC} = \frac{JC_p T_1 (\tau^{.5} - 1)}{\eta_c \eta_{sc} \frac{U^2}{g_o} \frac{C_a}{U} (\tan \beta_2 - \tan \beta_1)} \qquad \text{(B-18)}$$

where η_{sc} is the stage efficiency. (B-18) is in the form whereby mach number considerations might dictate the number of stages since

$$\frac{T_1}{U^2} = \frac{KRg_oT_1}{KRg_oU^2} = \frac{1}{KRg_oM^2} \quad .$$

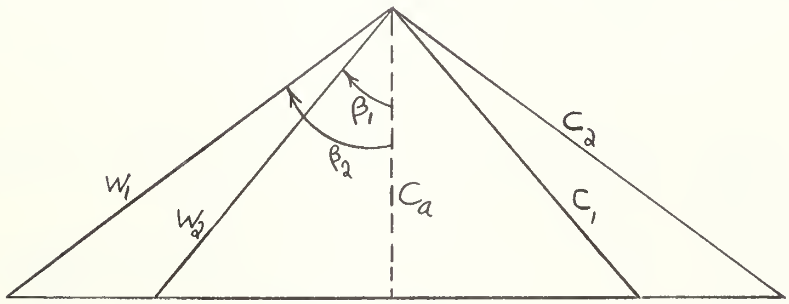
By substituting for U^2 from equation (B-4), (B-18) may be put in the form:

$$N_{LPC} = \frac{2JC_p T_1 (\tau^{.5} - 1)\ (\ell/d)_1}{\eta_c \eta_{sc} \beta\ \frac{C_a}{U} (\tan \beta_2 - \tan \beta_1)} \qquad \text{(B-19)}$$

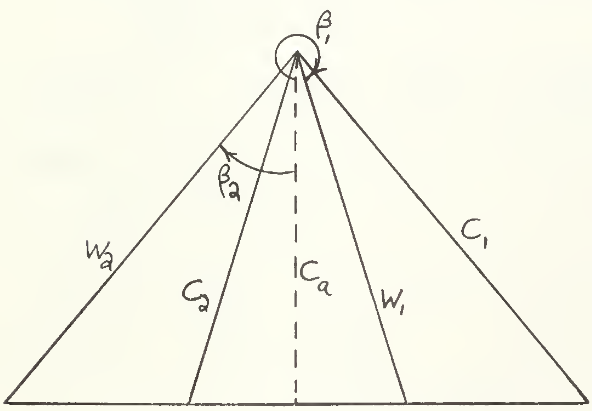
In (B-19), stress considerations now influence the number of stages.

For the HPC an expression similar to (B-19) is applicable since the inlet temperature to both compressors

is equal, and only the $\frac{\ell}{d}$ ratio and the angles β_2 and β_1 may vary slightly from that of the LPC.



Compressor



Turbine

Figure B-1

Stage Velocity Diagram

III. Turbines

Before developing the corresponding equations for the turbines, it is necessary to determine the pressure ratio across each turbine as a function of the overall cycle pressure ratio. Since the HPT drives only the two compressors, the work of the HPT must equal the total compressor work:

$$W_{HPT} = \eta_t C_p (T_6 - T_{7s}) = \eta_t C_p T_6 \left(1 - \frac{1}{\tau_{HPT}}\right) \tag{B-20}$$

From equation (2-2):

$$W_C = \frac{2\ C_p T_1}{\eta_c} (\tau^{.5} - 1) \tag{B-21}$$

Equating (B-20) and (B-21) and solving for τ_{HPT}:

$$\tau_{HPT} = \frac{1}{1 - \frac{2T_1}{T_6 \eta_t \eta_c} (\tau^{.5} - 1)} = \frac{1}{1 - C(\tau^{.5} - 1)} \tag{B-22}$$

where C is a constant since T_1, T_6, η_t and η_c will be fixed for a given cycle.

From equation (2-4):

$$(\tau_{HPT})(\tau_{LPT}) = \tau_t = Y\tau \tag{B-23}$$

Substituting (B-22) into (B-23):

$$\tau_{LPT} = Y\tau[1 - C(\tau^{.5} - 1)] \tag{B-24}$$

At the HPT exit (LPT inlet) the temperature can be expressed as a function of T_6 and pressure ratio. Using the definition of turbine work and (B-22):

$$T_7 = T_6\ [1 - \eta_t C(\tau^{.5} - 1)] \tag{B-25}$$

Following the procedure used for the compressors

and the preceding work in this section, the pitch diameter and annulus area at the HPT exit can be written:

$$d_{HPT} = K\left(\frac{\dot{w}T_6[1-\eta_t C(\tau^{.5}-1)]}{P[1-C(\tau^{.5}-1)]^u}\right)^{.5}\left(\frac{1}{C_{a/U}}\right)^{.5}\left(\beta\frac{\ell}{d}\right)^{-.25} \qquad \text{(B-26)}$$

where $C = \dfrac{2T_1}{T_6\eta_t\eta_c}$

$$A_{HPT} = K\,\frac{\dot{w}T_6[1-\eta_t C(\tau^{.5}-1)]}{P[1-C(\tau^{.5}-1)]^u}\quad\frac{1}{C_{a/U}}\,(\beta)^{-.5}\left(\frac{\ell}{d}\right)^{.5} \qquad \text{(B-27)}$$

For the low pressure turbine:

$$d_{LPT} = K\left(\frac{\dot{w}T_6[1-\eta_t(1-\frac{1}{Y\tau})]r}{P}\right)^{.5}\left(\frac{1}{C_{a/U}}\right)^{.5}\left(\beta\,\frac{\ell}{d}\right)^{-.25} \qquad \text{(B-28)}$$

$$A_{LPT} = K\,\frac{wT_6[1-\eta_t(1-\frac{1}{Y\tau})]r}{P}\quad\left(\frac{1}{C_{a/U}}\right)(\beta)^{-.5}\left(\frac{\ell}{d}\right)^{.5} \qquad \text{(B-29)}$$

The tip diameter at the exhaust end of the LPT is given by:

$$D_t = \left(1+\left(\frac{\ell}{d}\right)_8\right)d_{LPT} \qquad \text{(B-30)}$$

For the HPT, the number of stages is:

$$N_{HPT} = \frac{JCpT_6\left(1-\frac{1}{\tau_{HPT}}\right)\eta_t\eta_{st}}{\frac{U^2}{g_o}\quad\frac{C_a}{U}(\tan\beta_2-\tan\beta_1)} \qquad \text{(B-31)}$$

With (B-22) substituted for τ_{HPT}, (B-31) becomes:

$$N_{HPT} = \frac{2JCpT_6C(\tau^{.5}-1)\eta_t\eta_{st}}{\beta\,\frac{C_a}{U}\,(\tan\beta_2-\tan\beta_1)}\left(\frac{\ell}{d}\right)_7 \qquad \text{(B-32)}$$

where (B-4) was used to substitute for U^2 in (B-31), C is again the term defined with (B-22).

For the LPT, the number of stages become:

$$N_{LPT} = \frac{JC_p T_7 \left(1 - \frac{1}{\tau_{LPT}}\right) \eta_t \eta_{st}}{\frac{U^2}{g_o} \frac{C_a}{U} (\tan \beta_2 - \tan \beta_1)} \qquad \text{(B-33)}$$

Using (B-4), (B-24), and (B-25):

$$N_{LPT} = \frac{2JC_p T_6 [1-\eta_t C(\tau^{.5}-1)][Y\tau(1-C[\tau^{.5}-1])-1]\eta_t \eta_{st} \left(\frac{\ell}{d}\right)_8}{\beta \frac{C_a}{U} (\tan \beta_2 - \tan \beta_1) Y\tau[1-C(\tau^{.5}-1)]} \qquad \text{(B-34)}$$

The turbo-machinery equations developed in this and the preceding section appear to be quite cumbersome. However, it should be noted that many of the factors (efficiencies, $\frac{\ell}{d}$, $\frac{C_a}{U}$, β, β_2 and β_1) will be fixed by the turbine and compressor design, and will be applicable to a wide range of operating conditions. Then the only variables left to the marine engineer's discretion are T_6, T_1, τ or r, Y and volumetric flow rate.

APPENDIX C

EVALUATION OF CONSTANTS

I. General

The MGCR reference plant has a reactor rating of 49.7 MW (thermal) with a flow rate of 74.6 lb/sec. The coolant top and low temperatures in the cycle are respectively:

$$T_6 = 1300^{\circ}F \text{ and } T_1 = 100^{\circ}F.$$

Reactor inlet temperature is $T_5 = 791^{\circ}F$. The ratio of turbine pressure ratio to compressor pressure ratio is 0.86, giving a pressure drop of 140 psi through the cycle. Compressor pressure ratio is 2.6 with the top pressure of the fluid at 1000 psia. Turbine and compressor efficiencies are respectively:

$$\eta_t = 0.90 \text{ and } \eta_c = .86.$$

With the above conditions, the LPT produces 20,600 HP, and the HPT 746 auxiliary horsepower in addition to driving the compressor unit. In this work, the HPT provides only sufficient power to overcome the compressor work, leaving the LPT with an output of 21,500 HP for the above conditions.

The remaining temperatures of the reference cycle are given below. The state points correspond to those indicated in Figure II-1.

$T_1 = T_3 = 100^\circ F$ $\qquad$ $T_8 = 852^\circ F$

$T_2 = T_4 = 239^\circ F$ $\qquad$ $T_9 = 301^\circ F$

$T_5 = 791^\circ F$ $\qquad$ $T_{w1} = 70^\circ F$

$T_6 = 1300^\circ F$ $\qquad$ $T_{w2} = 90^\circ F$

$T_7 = 1008^\circ F$

II. Regenerator

The regenerator is composed of three equivalent units arranged in series. Since no heavy internal supporting structures are necessary, the regenerator can be constructed in a U-shaped fashion to conserve space when a high regenerator effectiveness is needed. This shape unit would also simplify the thermal expansion problems. The constants evaluated for the regenerator dimensions and weight are based on the overall size and weight of the regenerator. Unit length and weight can then be found by dividing by 3. Each unit uses 3/8 inch outside diameter tubes, 20BWG, with a 5/8 inch center-to-center triangular tube spacing. Heat transfer and cross section area and effective tube length are:

$$NdL = 23{,}340 \text{ ft}^2$$

$$Nd^2 = 7.1 \text{ ft}^2$$

$$L = 81 \text{ ft}$$

Total weight of the reference regenerator having an effectiveness of 0.90 is:

$$WT = 37.1 \text{ tons}$$

For the above tube data, q as given by equation (A-9A) in Appendix A is equal to 2.06. With the above information, equation (2-9) is written:

$$Nd^2 = K_{ND}\, w\left(\frac{\eta_x}{\eta_x}\right)^{.5} \frac{1}{P}\left(T_c + .0421(r)^2\ T_h\right)^{.5} \tag{C-1}$$

with T_c and T_h defined by (2-11) and (2-12) respectively. Using (C-1) and the above information K_{ND} is evaluated as:

$$K_{ND} = .91 \tag{C-2}$$

Upon using (2-10), K_{NDL} becomes:

$$K_{NDL} = 223.5 \tag{C-3}$$

Number of tubes is given by:

$$N = \frac{\text{cross sectional area}}{\frac{\pi}{4}\, d_o^2\ (q + 1)} = \frac{Nd^2}{\frac{\pi}{4}\, d_o^2\ (q + 1)} \tag{C-4}$$

where the denominator equals 2.35×10^{-3} for the given tube data.

Effective tube length can be found by using:

$$L = \frac{\text{heat transfer area}}{\pi\ N\ d_o} = \frac{NdL}{\pi\ N\ d_o} \tag{C-5}$$

Total regenerator weight as given by (2-16) is:

$$W_{RG} = K_1(NdL) + K_2$$

where NdL is equation (2-10). Assuming headers and support equipment weigh 3 tons per unit, K_2 is taken as:

$$K_2 = 9.0 \text{ tons} \qquad (C\text{-}6)$$

Using (2-16), (C-3) and the given operating conditions, K_1 is then evaluated as:

$$K_1 = 1.21 \times 10^{-3} \text{ tons/ft}^2 \qquad (C\text{-}7)$$

Cross sectional and heat transfer area and the regenerator weight can then be written as:

$$Nd^2 = .91(w)\left(\frac{\eta_x}{1-\eta_x}\right)^{.5} \frac{1}{P} (T_c + .0421(r)^2 \, T_h)^{.5} \qquad (C\text{-}8)$$

$$NdL = 223.5 \, w\left(\frac{\eta_x}{1-\eta_x}\right)^{1.4} \frac{1}{P^{.8}} (T_c + .0421(r)^2 T_h)^{.4} \qquad (C\text{-}9)$$

$$W_{RG} = 1.21 \times 10^{-3}(NdL) + 9 \qquad (C\text{-}10)$$

with NdL given by (C-9).

III. <u>Coolers</u>

Both coolers are similar in arrangement and construction. Each uses 5/8 inch outside diameter tubes, 18 BWG, with a 7/8 inch center-to-center triangular tube spacing. Weight, effective tube length and cross sectional and heat transfer area for the precooler used in

the reference plant are respectively:

$$W_{pc} = 13.6 \text{ tons}$$

$$L = 19.2 \text{ ft}$$

$$Nd^2 = 11.6 \text{ ft}^2$$

$$NdL = 7600 \text{ ft}^2$$

Using equation (2-17), the above information and the conditions stated in part I of this appendix, K_{ND} is evaluated as

$$K_{ND} = 1.48 \tag{C-11}$$

With (2-18), K_{NDL} becomes:

$$K_{NDL} = 290 \tag{C-12}$$

Number of tubes and length are given by expressions similar to (C-4) and (C-5) respectively.

Precooler weight is found by using (2-35):

$$W_{pc} = K_1 w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{1.4} \left(T_{pc}\right)^{.4} \left(\frac{r}{P}\right)^{.8} + K_2 \tag{2-35}$$

with ΔT_g and T_{pc} given by (2-19) and (2-20) respectively.

ΔT_m is the log mean temperature difference.

Assuming a header weight of 5 tons:

$$K_2 = 5 \text{ tons} \tag{C-13}$$

Using (2-35), (C-12), (C-13) and the given conditions,

K_1 is evaluated as:

$$K_1 = 1.13 \times 10^{-3} \text{ tons/ft}^2 \tag{C-14}$$

This value of K_1 is of the same order of magnitude as that for a steam condenser [9] but somewhat smaller.

Weight, effective tube length and cross sectional and heat transfer area for the intercooler of the reference design are:

W_{ic} = 13.4 tons

L = 14.0 ft

Nd^2 = 13.4 ft²

NdL = 6690 ft²

With (2-24) and the given conditions:

$$K_{ND} = 3.21 \tag{C-15}$$

Given conditions applied to (2-25) evaluates K_{NDL} as:

$$K_{NDL} = 477 \tag{C-16}$$

Equation (2-36) gives intercooler weight as:

$$W_{ic} = K_1 w \left(\frac{\triangle T_g}{\triangle T_m}\right)^{1.4} T_{ic}^{\ .4} \quad \frac{r^{.4}}{P^{.8}} + K_2 \tag{2-36}$$

K_2 for the intercooler is considered larger than that for the precooler because of the higher pressure associated with the intercooler.

$$K_2 = 6.5 \text{ tons} \tag{C-17}$$

Combining (2-36), (C-16), (C-17) and given conditions:

$$K_1 = 1.03 \times 10^{-3} \text{ tons/ft}^2 \qquad \text{(C-18)}$$

Summarizing, the equations for the precooler and intercooler are:

Precooler:

$$Nd^2 = 1.48\, w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{.5} T_{pc}^{\ .5}\ \frac{r}{P} \qquad \text{(C-19)}$$

$$NdL = 290\, w \left(\frac{\Delta T}{\Delta T}\right)^{1.4} T_{pc}^{\ .4} \left(\frac{r}{P}\right)^{.8} \qquad \text{(C-20)}$$

$$W_{pc} = 1.13 \times 10^{-3}\,(NdL) + 5 \qquad \text{(C-21)}$$

with NdL given by (C-20).

Intercooler:

$$Nd^2 = 3.21\, w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{.5} T_{ic}^{\ .5}\ \frac{r^{.5}}{P} \qquad \text{(C-22)}$$

$$NdL = 477\, w \left(\frac{\Delta T_g}{\Delta T_m}\right)^{1.4} T_{ic}^{\ .4}\ \frac{r^{.4}}{P^{.8}} \qquad \text{(C-23)}$$

$$W_{ic} = 1.03 \times 10^{-3}\,(NdL) + 6.5 \qquad \text{(C-24)}$$

with NdL given by (C-23).

IV. Reactor

Core radius cubed for the reference reactor was developed in sub-section C of Section II and can be written from equation (2-52) as:

$$R^3 = K_R \frac{w}{P^{.8}} (T_6 - T_5)^{1.4} \tag{C-25}$$

With the core diameter of the MGCR reactor equal to 6.87 ft, and for the given operating conditions, K_R is evaluated from (C-25) as:

$$K_R = .0176 \tag{C-26}$$

The weight of the reference ractor is approximately 173 tons. Combining (2-51) and (2-53):

$$W_R = \rho_R \, 5.81 \, R^3 \tag{C-27}$$

Substituting (C-25) and (C-26) into (C-27) and applying the given conditions, reactor density is evaluated as:

$$\rho_R = .92 \text{ tons/(ft}^3 \text{ of reactor core)} \tag{C-28}$$

Considering the reactor vessel (exclusive of shielding), ρ_R = .105 tons/ft³. Substituting (C-25), (C-26) and (C-28) into (C-27), reactor weight becomes:

$$W_R = .0945 \, (T_6 - T_5)^{1.4} \frac{w}{P^{.8}} \tag{C-29}$$

Core radius cubed is given by:

$$R^3 = .0176 \frac{w}{P^{.8}} (T_6 - T_5)^{1.4} \tag{C-30}$$

Core height can then be found using (2-51):

$$H = 1.85R \tag{2-51}$$

Reactor vessel radius (R_V) and height H_V are given by (2-56) and (2-57):

$$R_V = 1.63\ (R) \tag{2-56}$$

$$H_V = 3.31\ (H) \tag{2-57}$$

V. <u>Turbo Machinery</u>

In developing the turbo machinery equations, a large number of variables are present. Fortunately most of these variables are a function of the machine design and not a strong function of the cycle operating conditions. Therefore most similar machine variables will have values that are near those indicated below (for the reference plant) for a considerable range of operating conditions.

<u>Both machines</u>:

$\beta \approx 10{,}000 - 12{,}000$ ft

$C_{a/U} \approx 1/2$

		Compressor	Turbine
$\frac{\ell}{d}$ (low pressure end)	$\approx$	.11	.20
efficiency	$\approx$	.86	.90

Weight, machine length and tip diameter of the rotating machinery are considered to be of significance in determining the overall plant size and weight. Therefore the constants associated with each of these three developed equations (weight, length and tip diameter) were

evaluated for each rotating machine.

The following data apply to the machinery used in the reference design:

	LPC	HPC	HPT	LPT
L (ft)	7.95	10.23	3.98	2.84
W (tons)	4.8	3.92	3.42	3.33
D (inches)	18.1	14.3	20.2	25.0

The applicable length (L), weight (W) and tip diameter (D) equations from sub-section D of Section II were used in conjunction with the above given geometry data and the operating conditions to evaluate the constants. The following values were obtained as constants:

	LPC	HPC	HPT	LPT
K_ℓ (ft)	37.0	47.6	18.5	27.2
K_w $\frac{\text{Lbf sec}}{\text{in}^2}$	113.0	151.0	156.0	216.0
$K_d \left(\frac{\text{sec Lbf}}{\text{Lbm}}\right)^{.5}$	40.7	41.1	63.4	65.3

For the LPC, the working equations are:

$$L = 37.0(\tau^{.5} - 1) \tag{C-31}$$

$$W = 113.0 \frac{wr}{P} (\tau^{.5} - 1) \tag{C-32}$$

$$D = 40.7 \left(\frac{wr}{P}\right)^{.5} \tag{C-33}$$

For the HPC:

$$L = 47.6(\tau^{.5} - 1) \tag{C-34}$$

$$W = 151.0 \frac{w}{P} r^{.5} (\tau^{.5} - 1) \tag{C-35}$$

$$D = 41.1 \left(\frac{w}{P}\right)^{.5} r^{.25} \tag{C-36}$$

For the HPT:

$$L = 18.5 (\tau^{.5} - 1) \tag{C-37}$$

$$W = 156 \frac{w}{P} \frac{(\tau^{.5}-1)[1-\eta_t C(\tau^{.5}-1)]}{[1 - C(\tau^{.5} - 1)]^u} \tag{C-38}$$

$$D = 63.4 \left(\frac{w}{P}\right)^{.5} \left(\frac{[1-\eta_t C(\tau^{.5}-1)]}{[1 - C(\tau^{.5}-1)]^u}\right)^{.5} \tag{C-39}$$

where $C = \frac{2T_1}{T_6 \eta_t \eta_c}$ and $u = \frac{K}{K-1}$

For the LPT:

$$L = \frac{27.2 [1-\eta_t C(\tau^{.5}-1)][Y\tau(1-C[\tau^{.5}-1])-1]}{Y\tau [1-C(\tau^{.5}-1)]} \tag{C-40}$$

$$W = 216.0 \frac{wr}{P} \frac{[1-\eta_t(1-\frac{1}{Y\tau})][1-\eta_t C(\tau^{.5}-1)][Y\tau(1-C[\tau^{.5}-1])-1]}{Y\tau[1 - C(\tau^{.5} - 1)]} \tag{C-41}$$

$$D = 65.3 \left(\frac{wr}{P}\right)^{.5} [1 - \eta_t (1 - \frac{1}{Y\tau})]^{.5} \tag{C-42}$$

APPENDIX D

SAMPLE CALCULATIONS

I. General

In order to indicate the effect of certain parameters as they individually influence plant efficiency, weight and space, it is desirable to fix as many other variables as possible. As stated in Section I, the top and low coolant temperatures, reactor inlet temperature, turbo-machinery efficiencies and pressure losses for the cycle will generally be fixed for a power plant of this type. From consideration of the developed equations, the remaining parameters effecting the cycle are pressure (P), flow rate (w), pressure ratio (r), and regenerator effectiveness (η_x). However, as noted in Section I, regenerator effectiveness is specified once a pressure ratio is selected for this type cycle.

Solution of all equations was performed for a LPT (net power) output of 21,500 HP with the below fixed conditions:

$T_6 = 1760^{\circ}R$	$\eta_t = .90$
$T_1 = 560^{\circ}R$	$\eta_c = .86$
$T_5 = 1251^{\circ}R$	
$P = 1000$ psia	$\frac{r_t}{r} = .86$ or $Y = .941$
$t_{w1} = 70^{\circ}F$	$t_{w2} = 90^{\circ}F$

Another set of calculations were made with the reactor inlet temperature lowered to 1160°R, with all other parameters above held constant.

Equations were evaluated for pressure ratios ranging from 1.3 to a ratio that placed the LPT exhaust temperature (T_8) at the same level as the reactor inlet temperature (T_5). This condition would require a regenerator effectiveness (η_x) of 100 %. Any pressure ratio larger than this value would present an impossible cycle since η_x would have to be greater than 100 %.

For equations where w and P appear, w is in lb/sec and P is in psia. All temperatures are in degrees absolute (°R).

II. Sample Calculations

Sample calculations are indicated for all equations using a pressure ratio of 2.4 and the fixed conditions given in Section I of this appendix.

A. Flow rate

Flow rate is found using equation (2-5):

$$\text{net power} = w\,[\eta_t + T_6 C_p (1 - \frac{1}{Y\tau}) - \frac{2\,T_1 C_p}{\eta_c} (\tau^{.5} - 1)]$$

$$\frac{(21,500)550}{778} = w[.90(1760)1.25\left(1 - \frac{1}{(.941)(1.42)}\right) - \frac{2(560)(1.25)}{.86}(1.19 - 1)]$$

$w = \underline{81.1 \text{ lb/sec}}$

$$w = \frac{(81.1)(3600)}{21,500} = \underline{13.6 \text{ lb}/(\text{HP}-\text{HR})}$$

B. Efficiency

Using (2-8), efficiency becomes

$$\eta = \frac{.90\,\frac{1760}{560}\left(1 - \frac{1}{(.941)(1.42)}\right) - \frac{2}{.86}(1.19 - 1)}{\frac{1760 - 1251}{560}}$$

$$\eta = \underline{0.294}$$

C. Cycle Temperatures

T_4 is found using the following equation obtained from compressor work:

$$T_4 = T_1\,[1 + \frac{1}{\eta_c}\,(\tau^{.5} - 1)]$$

$$T_4 = T_2 = 560\,[1 + \frac{1}{.86}\,(1.19 - 1)] = \underline{686^{\circ}\text{R}}$$

T_8 is given by:

$$T_8 = T_6\,[1 - \eta_t(1 - \frac{1}{Y\tau})]$$

$$= 1760\left[1 - .90\left(1 - \frac{1}{(.941)1.42}\right)\right]$$

$$T_8 = \underline{1362^{\circ}\text{R}}$$

T_9 is found from a heat balance around the regnerator:

$$T_5 - T_4 = T_8 - T_9$$

$$T_9 = 1362 - 1251 + 686$$

$$T_9 = \underline{797^{\circ}R}$$

D. Regenerator effectiveness

η_x is given by (A-4) in Appendix A:

$$\eta_x = \frac{T_5 - T_4}{T_8 - T_4}$$

$$\eta_x = \frac{1251 - 686}{1362 - 686}$$

$$\eta_x = \underline{0.836}$$

E. Regenerator weight and dimensions

$$T_c = \frac{T_5 + T_4}{2} = \frac{1251 + 686}{2} = 968^{\circ}R$$

$$T_h = \frac{T_8 + T_9}{2} = \frac{1362 + 797}{2} = \underline{1080^{\circ}R}$$

Putting numbers in (C-8), (C-9) and (C-10) respectively:

$$Nd^2 = .91(81.1)\left(\frac{.836}{1-.836}\right)^{.5} \frac{1}{1000} \left(968 + .0421(2.4)^2 1080\right)^{.5}$$

$$Nd^2 = \underline{5.85\ ft^2}$$

$$NdL = 223.5\,(81.1)\left(\frac{.836}{1-.836}\right)^{1.4} \left(\frac{1}{1000}\right)^{.8} \left(968 + .0421(2.4)^2 1080\right)^{.4}$$

$NdL = \underline{12,210 \text{ ft}^2}$

$W_{RG} = 1.21 \times 10^{-3} (12,210) + 9$

$W_{RG} = \underline{23.8 \text{ tons}}$

Using (C-4) and (C-5), the number of tubes and length are respectively:

$$N = \frac{5.85}{2.35 \times 10^{-3}} = \underline{2490 \text{ tubes}}$$

$$L = \frac{12,210}{(3.14)(2490)\left(\frac{3}{8}\right)\frac{1}{12}} = \underline{50 \text{ ft.}}$$

$$L = \frac{50}{3} = \underline{16.67 \text{ ft/unit.}}$$

F. Precooler weight and dimensions

$$T_{pc} = \frac{T_9 + T_1}{2} = \frac{797 + 560}{2} = \underline{678^\circ R}$$

$$\Delta T_g = T_9 - T_1 = 797 - 560 = \underline{237}$$

$$\Delta T_m = \frac{T_9 - T_{w2} - (T_1 - T_{w1})}{\ell n \frac{T_9 - T_{w2}}{T_1 - T_{w1}}} = \frac{797 - 550 - (560 - 530)}{\ell n \frac{247}{30}}$$

$$\Delta T_m = \underline{103^\circ F}$$

Using equations (C-19), (C-20) and (C-21)

$$Nd^2 = 1.48\ (81.1) \left(\frac{237}{103}\right)^{.5} (678)^{.5} \ \frac{2.4}{1000}$$

$$Nd^2 = \underline{11.38\ ft^2}$$

$$NdL = 290\ (81.1) \left(\frac{237}{103}\right)^{1.4} (678)^{.4} \left(\frac{2.4}{1000}\right)^{.8}$$

$$NdL = \underline{8220\ ft^2}$$

$$W_{pc} = 1.13 \times 10^{-3}\ (8220) + 5$$

$$W_{pc} = \underline{14.3\ tons}$$

$$N = \frac{11.38}{.00461}$$

$$N = \underline{2470\ tubes}$$

$$L = \frac{8220}{\pi\ (2470) \left(\frac{5}{8}\right) \frac{1}{12}}$$

$$L = \underline{20.4\ ft}$$

G. Intercooler weight and dimensions

$$T_{ic} = \frac{T_2 + T_1}{2} = \frac{686 + 560}{2} = \underline{623^{\circ}R}$$

$$\Delta T_g = T_2 - T_1 = 686 - 560 = \underline{126}$$

$$\Delta T_m = \frac{686 - 550 - (560 - 530)}{\ell n \dfrac{136}{30}}$$

$$\Delta T_m = \underline{70^{\circ}F}$$

Using (C-22) through (C-24):

$$Nd^2 = 3.21(81.1) \left(\frac{126}{70}\right)^{.5} (623)^{.5} \left(\frac{2.4}{1000}\right)^{.5}$$

$$Nd^2 = \underline{13.5 \text{ ft}^2}$$

$$NdL = 477\ (81.1) \left(\frac{126}{70}\right)^{1.4} (623)^{.4} \frac{(2.4)^{.4}}{(1000)^{.8}}$$

$$NdL = \underline{6590 \text{ ft}^2}$$

$$W_{ic} = 1.03 \times 10^{-3} (6590) + 6.5$$

$$W_{ic} = \underline{13.3 \text{ tons}}$$

$$N = \frac{13.5}{.00461} = \underline{2940 \text{ tubes}}$$

$$L = \frac{6590}{\pi(2940)\ \frac{5}{8}\ \frac{1}{12}}$$

$$L = \underline{13.7 \text{ ft}}$$

H. Reactor weight and dimensions

Using (C-29) and (C-30):

$$W_R = .0945\ (1760 - 1251)^{1.4} \ \frac{81.1}{(1000)^{.8}}$$

$$W_R = \underline{188 \text{ tons}}$$

$$R^3 = .0176 \frac{(81.1)}{(1000)^{.8}} (1760 - 1251)^{1.4}$$

$$R^3 = 35.04$$

$$R = \underline{3.28 \text{ ft}} \text{ (core radius)}$$

From (2-51) core height is:

$$H = 1.85 (3.28)$$

$$H = \underline{6.06 \text{ ft}} \quad \text{(core height)}$$

Reactor vessel radius and height are obtained from (2-56) and (2-57) respectively:

$$R_V = 1.63 (3.28)$$

$$R_V = \underline{5.35 \text{ ft}}$$

$$H_V = 3.31 (6.06)$$

$$H_V = \underline{20 \text{ ft}}$$

I. Rotating machinery

Using equations (C-31) through (C-42), the length, weight and tip diameter are given respectively for each unit:

<u>LPC</u>:

$$L = 37 (1.19-1)$$

$$L = \underline{7.0 \text{ ft}}$$

$$W = 113 \frac{(81.1)(2.4)}{1000} (1.19-1)$$

W = 4.2 tons

$$D = 4.07 \left(\frac{81.1\ (2.4)}{1000}\right)^{.5}$$

D = 18 inches

HPC:

$$L = 47.6\ (1.19-1)$$

L = 9.1 ft.

$$W = 151\ \frac{81.1}{1000}\ (2.4)^{.5}\ (1.19-1)$$

W = 3.61 tons

$$D = 41.1 \left(\frac{81.1}{1000}\right)^{.5} (2.4)^{.25}$$

D = 14.5 inches

HPT:

$$L = 18.5\ (1.19 - 1)$$

L = 3.52 ft.

$$W = 156\ \frac{81.1}{1000}\ \frac{(1.19-1)[1-.9(.822)(1.19-1)]}{[1-.822(1.19-1)]^{2.5}}$$

W = 3.2 tons

$$D = 63.4 \left(\frac{81.1}{1000}\right)^{.5} \left(\frac{[1-.9(.822)(1.19-1)]}{[1-.822(1.19-1)]^{2.5}}\right)^{.5}$$

D = 20.7 inches

LPT:

$$L = 27.2 \frac{\left[1-.9(.822)(1.19-1)\right]\left[.941(1.42)(1-.822[1.19-1])\right]}{.941(1.42)[1-.822(1.19-1)]}$$

L = 2.8 ft

$$W = 216 \quad \frac{81.1(2.4)}{1000}$$

$$\frac{\left[1-.9(1-\frac{1}{.941(1.42)})\right]\left[1-.9(.822)(1.19-1)\right]\left[.941(1.42)(1-.822[1.19-1]\right]}{.941(1.42)(1-.822[1.19-1])}$$

W = 3.2 tons

$$D = 65.3 \left(\frac{(81.1)2.4}{1000}\right)^{.5} \left(1 - .9 \left[1 - \frac{1}{.941(1.42)}\right]\right)^{.5}$$

D = 25.4 inches

APPENDIX E

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III. Special References

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18. MIT Course Notes, Nuclear Engineering (22.23)